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Numerical technique for solving First-Order System Ordinary Differential Equations

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ABSTRACT

In this article, a numerical method is presented to find a numerical solution (NS) of systems of ordinary differential equations (SODEs) by using the Differential Transform Method (DTM). Different kinds of linear systems of ordinary differential equations have been solved using the presented method with the initial conditions; at any point, this method transformed the SODEs into a series. Then it solved by using software MATLAB. Two different examples have been presented to show the efficiency and activity of the method. The solutions of the technique method outperformed the other methods in terms of accuracy and quick convergence.

الخلاصة

في هذا البحث، تم تقديم طريقة عددية لإيجاد حل عددي لنظام المعادلات التفاضلية الاعتيادية باستخدام طريقة تحويل المشتقة. وقد تم حل أنواع مختلفة من أنظمة المعادلات التفاضلية الاعتيادية الخطية باستخدام هذه الطريقة مع الشروط الابتدائية، حيث تُحوّل هذه الطريقة المعادلات التفاضلية الاعتيادية إلى متسلسلة في أي نقطة. ثم يتم حلها باستخدام برنامج MATLAB وقد عُرض مثالان مختلفان لإظهار كفاءة الطريقة وفعاليتها. وقد تفوقت حلول هذه الطريقة على الطرق الأخرى من حيث الدقة وسرعة التقارب.

1. Introduction

The DTM is a highly accurate approximation to exact solutions that are differentiable. When compared to exact solutions, it has a small error. Naturally, the DTM differs from conventional high-order Taylor series (TS) since it needs more computation, and it also requires the necessary derivatives [1]. Further, the DTM can also be applied to higher-order differential equations (DEs), and it is a substitute way to get TS solutions for the given DEs, for example [2-4].

Furthermore, applied DTM to find NS of partial differential equations (PDEs) in two dimensions in [5] and [6], and DTM was also used to find NS of PDEs in two dimensions and in three dimensions. Recently, [7-10] used DTM to solve a system of PDEs. All the above methods motivated us to think about the study of the NS by using the presented method, DTM.

This article begins with the description of the SODEs; the NS is obtained by using DTM, and at any point this method transforms the SODEs into a series. It solved by using software MATLAB.

2. Numerical method DTM

The transformation of the σth derivative of a function in on offered is a follow:

$$\gamma(\sigma) = \frac{1}{\sigma!}\left[\frac{d^{\sigma}\vartheta}{d\tau^{\sigma}}(\tau)\right]_{\tau=\tau_0} \qquad (1)$$

And the inverse transformation is well-defined by,

$$\vartheta(\tau) = \sum_{\sigma=0}^{\infty}\gamma(\sigma)(\tau-\tau_0) \qquad (2)$$

See [8]

"The following theorems that can be deduced from equations (1) and (2) are given below":

Theorem (1). If $\vartheta(\tau) = \vartheta_1(\tau) \pm \vartheta_2(\tau)$, then$\gamma_1(\sigma) \pm \gamma_2(\sigma)$.

Theorem (2). If $\vartheta(\tau) = \mu\vartheta_1(\tau)$, then $\gamma(\sigma) = \mu\gamma_1(\sigma)$, where$\mu$ is a constant.

Theorem (3). If $\vartheta(\tau) = \frac{d^n\vartheta_1(\tau)}{d\tau^n}$, then$\gamma(\sigma) = \frac{(\sigma+n)!}{\sigma!}\gamma_1(\sigma+n)$.

Theorem (4). If $\vartheta(\tau) = \vartheta_1(\tau)\vartheta_2(\tau)$, then $\gamma(\sigma) = \sum_{\sigma_1}^{\sigma}\gamma_1(\sigma_1)\gamma_2(\sigma-\sigma_1)$.

Theorem (5). If $\vartheta(\tau) = \tau^n$, then $\gamma(\sigma) = \beta(\sigma-n)$ where $\beta(\sigma-n) = \begin{cases}1 & \sigma = n\\ 0 & \sigma \neq n\end{cases}$.

Theorem (6). If $\vartheta(\tau) = \vartheta_1(\tau)\vartheta_2(\tau)\ldots\vartheta_n(\tau)$, then $\gamma(\sigma) = \sum_{\sigma_{n-1}=0}^{\sigma}\sum_{\sigma_{n-2}=0}^{\sigma_{n-1}}\ldots\sum_{\sigma_2=0}^{\sigma_3}\sum_{\sigma_1=0}^{\sigma_2}\gamma_1(\sigma_1)\gamma_2(\sigma_2-\sigma_1)\ldots\vartheta_{n-1}(\sigma_{n-1}-\sigma_{n-2})\gamma_n(\sigma-\sigma_{n-1})$.

Theorem (7). If $\vartheta(\tau) = e^{\lambda\tau}$, then $(\sigma) = \frac{\lambda^{\sigma}}{\sigma!}$, where λ is a constant.

Theorem (8). If $\vartheta(\tau)=\sin(\omega\tau+\alpha)$, then $\gamma(\sigma)=\frac{\omega^\sigma}{\sigma!}\sin\left(\frac{\sigma\pi}{2}+\alpha\right)$, where ω and α is constant.

Theorem (9). If $\vartheta(\tau)=\cos(\omega\tau+\alpha)$, then $\gamma(\sigma)=\frac{\omega^\sigma}{\sigma!}\cos\left(\frac{\sigma\pi}{2}+\alpha\right)$, when ω and α constants.

The proofs of Theorems (1-6) are offered in [2], and the proofs of Theorems (7-9) are offered in [10].

3. Numerical results

We present some Applications which are solved using DTM.

Application 3.1

Solve the homogeneous linear system.

$$\frac{d\vartheta_1}{d\tau}-\vartheta_2(t)=0 \tag{3}$$

$$\frac{d\vartheta_2}{d\tau}+\vartheta_1(t)=0 \tag{4}$$

Subject to the initial condition

(IC) $\vartheta_1(0)=1,\vartheta_2(0)=0$ (5)

The EXS of this problem is:

$$\vartheta_1(\tau)=\cos(\tau)\,,\vartheta_2(\tau)=-\sin(\tau) \tag{6}$$

“Taking the DTM to equations (3) and (4), applying the above-mentioned, we obtain”

$$(\sigma+1)\gamma_1(\sigma+1)-\gamma_2(\sigma)=0 \tag{7}$$

$$(\sigma+1)\gamma_2(\sigma+1)+\gamma_1(k)=0 \tag{8}$$

With initial conditions (ICs) $\gamma_1(0)=1,\gamma_2(0)=0$

Let σ=0,$\gamma_1(1)=0,\gamma_2(1)=-1$

σ=1, $\gamma_1(2)=\frac{-1}{2},\gamma_2(2)=0$

σ=2, $\gamma_1(3)=0,\gamma_2(3)=\frac{1}{6}$

σ=3, $\gamma_1(4)=\frac{1}{24},\gamma_2(4)=0$

σ=4, $\gamma_1(5)=0,\gamma_2(5)=\frac{-1}{120}$

σ=5, $\gamma_1(6)=\frac{-1}{720},\gamma_2(6)=0$

(So the solution when n=6)

$\vartheta_i(\tau)=\sum_{\sigma=0}^{6}\tau^\sigma\gamma_i(\sigma)$ For i=1.2

$$\vartheta_1(\tau)=1-\frac{1}{2!}\tau^2+\frac{1}{4!}\tau^4-\frac{1}{6!}\tau^6$$

$$\vartheta_2(\tau)=-\tau+\frac{1}{3!}\tau^3-\frac{1}{5!}\tau^5=-\left(\tau-\frac{1}{3!}\tau^3+\frac{1}{5!}\tau^5\right)$$

The results are shown in Table 1 and Figures 1 and 2 below.

Table 1: The results of Application (1)

τ	Exact $\vartheta_1(\tau)$	Exact $\vartheta_2(\tau)$	Approximate $\vartheta_1(\tau)$	Approximate $\vartheta_2(\tau)$	Error of $\vartheta_1(\tau)$	Error of $\vartheta_2(\tau)$
0	1.0000	0	1.0000	0	0.0000	0
0.1	0.9950	-0.0998	0.9950	-0.0998	0.0000	0.0000
0.2	0.9801	-0.1987	0.9801	-0.1987	0.0000	0.0000
0.3	0.9553	-0.2955	0.9553	-0.2955	0.0000	0.0000
0.4	0.9211	-0.3894	0.9211	-0.3894	0.0000	0.0000
0.5	0.8776	-0.4794	0.8776	-0.4794	0.0000	0.0000
0.6	0.8253	-0.5646	0.8253	-0.5646	0.0000	0.0000
0.7	0.7648	-0.6442	0.7648	-0.6442	0.0000	0.0000
0.8	0.6967	-0.7174	0.6967	-0.7174	0.0000	0.0000
0.9	0.6216	-0.7833	0.6216	-0.7834	0.0000	0.0001
1	0.5403	-0.8415	0.5403	-0.8416	0.0000	0.0001

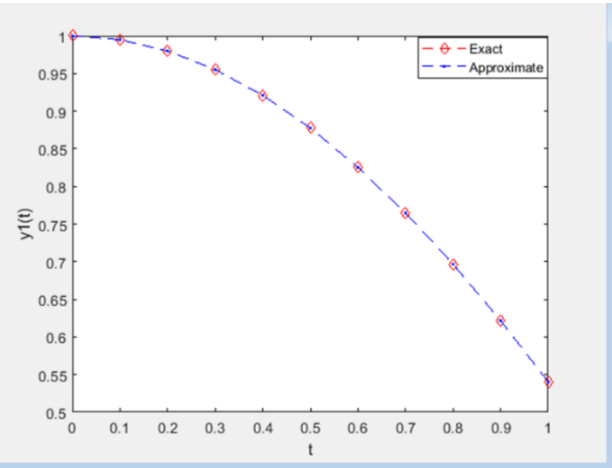

Figure1: Exact and Numerical solution for Application1

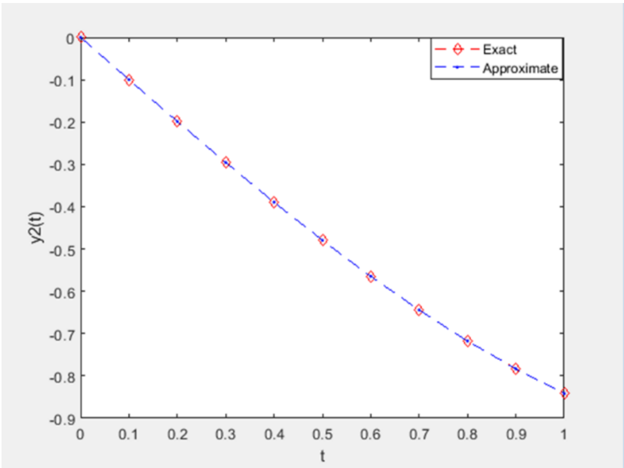

Figure2: Exact and Numerical solution for Application1

Application 3.2

Solve the non-homogeneous linear system [8]

$$\frac{d\vartheta_1}{d\tau}-\vartheta_3(\tau)=-\cos(\tau) \tag{9}$$

$$\frac{d\vartheta_2}{d\tau}-\vartheta_3(\tau)=-e^\tau \tag{10}$$

$$\frac{d\vartheta_3}{d\tau}-\vartheta_1(\tau)+\vartheta_2(\tau)=0 \tag{11}$$

Subject to (ICs) $\vartheta_1(0)=1,\vartheta_2(0)=0,\vartheta_3(0)=2$ (12)

The EXS of this problem is

$\vartheta_1(\tau) = e^{\tau}, \vartheta_2(\tau) = \sin(\tau), \vartheta_3(\tau) = e^{\tau} + \cos(\tau)$ (13)

"Taking the DTM to equations (9), (10), and (11), and applying the above-mentioned, we obtain"

Let σ=0, $\gamma_1(1) = 1, \gamma_2(1) = 1, \gamma_3(1) = 1$

σ=1, $\gamma_1(2) = \frac{1}{2}, \gamma_2(2) = 0, \gamma_3(2) = 0$

σ=2, $\gamma_1(3) = \frac{1}{6}, \gamma_2(3) = \frac{-1}{6}, \gamma_3(3) = \frac{1}{6}$

σ=3, $\gamma_1(4) = \frac{1}{24}, \gamma_2(4) = 0, \gamma_3(4) = \frac{1}{12}$

σ=4, $\gamma_1(5) = \frac{1}{120}, \gamma_2(5) = \frac{1}{120}, \gamma_3(5) = \frac{1}{120}$

(So the solution when n=5)

$\vartheta_i(\tau) = \sum_{\sigma=0}^{5} \tau^{\sigma} \gamma_i(\sigma)$ For i=1, 2, 3

$\vartheta_1(\tau) = 1 + \tau + \frac{1}{2!}\tau^2 + \frac{1}{3!}\tau^3 + \frac{1}{4!}\tau^4 + \frac{1}{5!}\tau^5$

$\vartheta_2(\tau) = \tau - \frac{1}{3!}\tau^3 + \frac{1}{5!}\tau^5$

$\vartheta_3(\tau) = \left(1 + \tau + \frac{1}{2!}\tau^2 + \frac{1}{3!}\tau^3\right) + (1 - \frac{1}{2!}\tau^2 + \frac{1}{4!}\tau^4)$

The results are shown in Table 2 and Figures 3-5 below.

Table (2): The results of Application (2)

τ	Exact $\vartheta_1(\tau)$	Exact $\vartheta_2(\tau)$	Exact $\vartheta_3(\tau)$	Approximate of $\vartheta_1(\tau)$	Approximate of $\vartheta_2(\tau)$	Approximate of ϑ_3 (τ)	Error of $\vartheta_1(\tau)$	Error of ϑ_2 (τ)	Error of $\vartheta_3(\tau)$
0	1.0000	0	2.0000	1.0000	0	2.0000	0.0000	0	0.0000
0.1	1.1052	0.0998	2.1002	1.1052	0.0998	2.1002	0.0000	0.0000	0.0000
0.2	1.2214	0.1987	2.2015	1.2214	0.1987	2.2014	0.0000	0.0000	0.0001
0.3	1.3499	0.2955	2.3052	1.3499	0.2955	2.3048	0.0000	0.0000	0.0004
0.4	1.4918	0.3894	2.4129	1.4918	0.3894	2.4117	0.0000	0.0000	0.0012
0.5	1.6487	0.4794	2.5263	1.6487	0.4794	2.5234	0.0000	0.0000	0.0029
0.6	1.8221	0.5646	2.6475	1.8221	0.5646	2.6414	0.0001	0.0000	0.0061
0.7	2.0138	0.6442	2.7786	2.0136	0.6442	2.7672	0.0002	0.0000	0.0114
0.8	2.2255	0.7174	2.9222	2.2252	0.7174	2.9024	0.0004	0.0000	0.0198
0.9	2.4526	0.7833	3.0812	2.4588	0.7834	3.0489	0.0008	0.0001	0.0323
1	2.7183	0.8415	3.2586	2.7167	0.8416	3.2084	0.0016	0.0001	0.0502

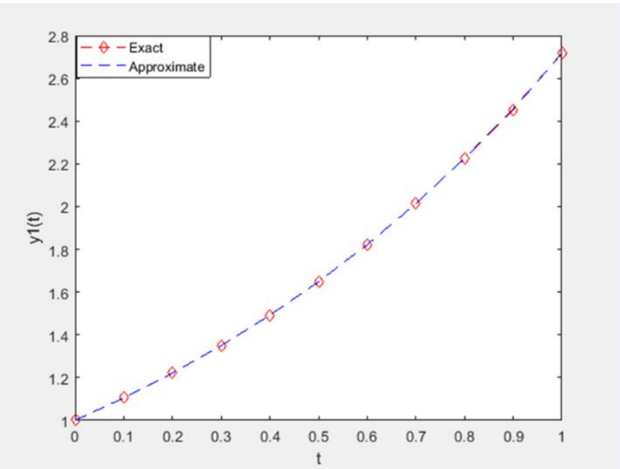

Figure 3: Exact and Numerical solution for Application2

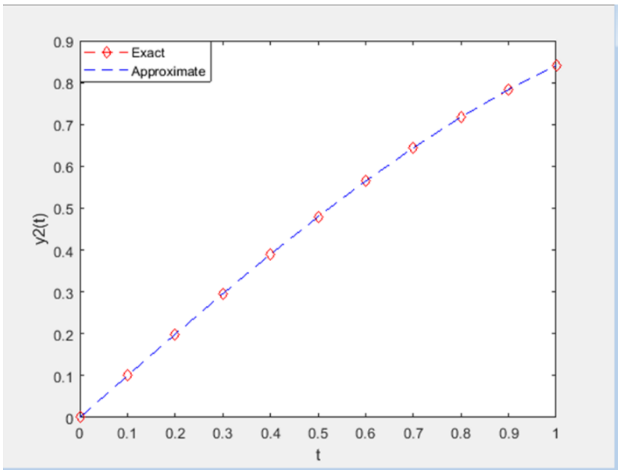

Figure 4: Exact and Numerical solution for Application2

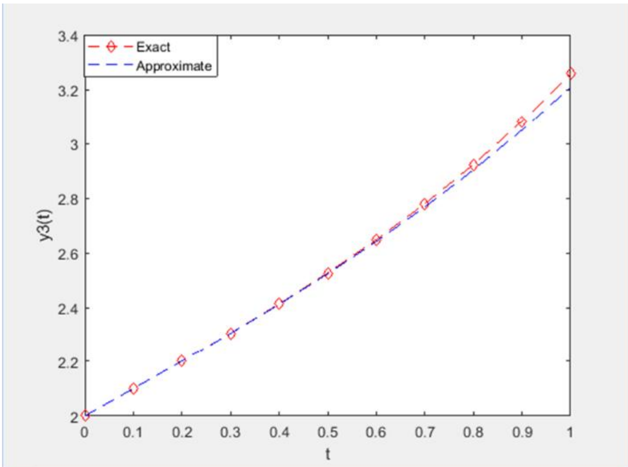

Figure 5: Exact and Numerical solution for Application2

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4. Conclusion

In this paper, the DTM is used for solving the SODEs numerically. Two numerical applications have been used to prove the efficiency and activity of the presented method. This method can be extended and applied to solve the Fredholm integral equations, Volterra integral equations, mixed Fredholm-Volterra integral equations, and system integral equations.

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