Determination of velocity, pressure and heat transfer of a steady state flow of third order fluid using homotopy analysis method

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Abstract:

This paper, focused on a steady flow of non-Newtonian fluid of three order in a porous walls vertical channel. The equations were used to describe it's state of motion and the energy equations. The velocity, pressure variations and heat transfer profiles investigated by using Homotopy Analysis Method and examining qualitatively the effect of non-Newtonian parameters (α, β, γ) which are dimensionless numbers, Reynolds number (Re), Hartmann number (M), and Peclet number (Pe) on these values. Finally varies numerical simulations were taken to enhance the analytical results.

Keywords: Third-order; Dimensionless numbers; Vertical channel; Velocity; pressure; numerical simulations.

تحديد السرعة والضغط وانتقال الحرارة لتدفق جريان منتظم لمائع من الدرجة الثالثة باستخدام طريقة هوموتوبي التحليلية

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الخلاصة:

ركز هذا البحث على التدفق المنتظم للسائل غير النيوتوني من الرتبة الثالثة في قناة عمودية ذات جدران مسامية. تم استخدام المعادلات لوصف حالة الحركة ومعادلات الطاقة. تم فحص السرعة وتغيرات الضغط وانتقال الحرارة باستخدام طريقة تحليل Homotopy والفحص النوعي لتأثير المعلمات غير النيوتينية (α، β ،) وهي أرقام بلا أبعاد ، رقم رينولدز (Re) ، رقم هارتمان (M) ، و Peclet رقم (Pe) على هذه القيم. أخيرًا تم إجراء عمليات محاكاة عددية مختلفة لتعزيز النتائج التحليلية.

كلمات مفتاحية : رتبة ثالثة– اعداد لا بعدية –قناة عمودية–سرعة – ضغط– محاكاة عددية.

1.Introduction:

A lot of practical applications in real life are depending on fluid mechanics which is an interesting and exciting subject. These applications ranging from spacecraft, planes, vehicles to microscopic systems, unlike the other study materials of freshman and sophomore-level such as mechanics engineering, and physics. The flow curve of non-Newtonian fluid is (shear stress vs strain) non-linear or does not pass across the origin, in other words, shear stress divided by strain, isn't steady at a provided pressure and temperature but it's reliant on conditions of flow like a shear rate, flow geometry, etc. and in some cases even on the history of kinematic of the fluid component under consideration. Such materials could be suitably assembled into three common classes. The flow across the medium of porous has uncounted applications in engineering and science like chemical reactors, drainage, petroleum, chemical engineering, and irrigation [17]. The first Magnetohydrodynamics (MHD) Bingham plastic and power law fluids study was introduced by[1]. A fascinating study for the Maxwell fluid of hydrodynamic in a channel of porous was presented in[2]. MHD is one of the important study areas that has raised in the engineering sciences in the 21st century. MHD includes electrically conducting fluids that interact with magnetic fields.[3]solved the transformations presented by substituting thermodynamically the second grade compatible fluid instead of a Newtonian fluid. Baris used an old analytical method which is a conventional perturbation solution.[4,5] discussed the third-grade unsteady flow in a porous space. They utilized the modified Darcy's law in their fluid flow modelling study so,[6,7,8] have studied the transfer of heat through an unsteady expanding permeable surface with specified wall temperature. They applied the Homotopy Analysis Method (HAM) to achieve an analytic solution. HAM was first proposed by[9]. [11] ,[16]have studied the flow of thin-film unstable micropolar fluid over a medium of porous in the presence of MHD. To solve this problem, they utilized numerical techniques. They presented graphs for the effects of the various modelled parameters in their work.[10] have utilized HAM to find the heat transfer, velocity and pressure variation values and analytic the effect of non-dimensionless parameters with state flow in a Newtonian fluid have studied the solution of analytic for the flow of MHD third-order fluid and heat transfer through an expansion sheet. This paper is concerned with the formulation of the problems in which the governing nonlinear equations that describe the flowing nature are modelled. HAM is utilized to solve the problems under consideration. The implementation and results of this work are given by MATLAB®.

2. The Model:

The considered model for this paper is the third-order fluid steady in a vertical channel with porous walls. At y=D and with regular velocity U, a fluid is injected over a vertical pored plate. The fluid flowed through another vertical plate of impermeable at y=0. It is outflowing over the opening plates as the gravity action over the Z-axis. D is the distance between the walls and it's a small value against the plates dimensions as shown in Figure (1), i.e., L≫D. The impacts of the edge can be neglected [14].

The Cauchy tensor of stress in the case of third-order fluid is related to the equations of momentum in the following way [13]:

$$T = -PI + \mu A_1 + \alpha_1 A_2 + \alpha_2 {A_1}^2 + \beta_3\left(tras{A_1}^2\right)A_1 , \qquad (1)$$

where $A_1 = \nabla \boldsymbol{V} + (\nabla \boldsymbol{V})^T$,

$$A_2 = \frac{dA_1}{dt} + A_1(\nabla \boldsymbol{V}) + (\nabla \boldsymbol{V})^T A_1 , \qquad (2)$$

where $\mu, \alpha_1, \alpha_2, and\ \beta_3 \geq 0$. (3)

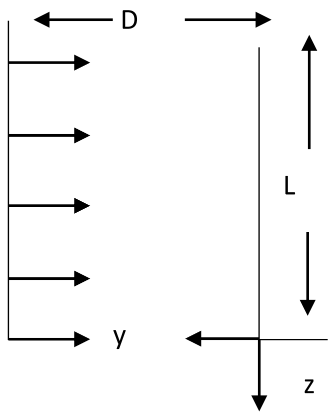

Fig. 1. Fluid flow in a vertical channel

where P represents the pressure, $\boldsymbol{V}$ represents the vector of velocity, ∇ is the operator of the gradient, $\alpha_i(i=1{,}2), (\beta_3)$ are the material moduli of fluid, *d/dt* is the material derivative, and $A_i(i=1{,}2)$ are the two first Rivilin Eriksen tensor. For $\alpha_1, \alpha_2 = 0$ and β_3 equation (1) along with (2) descries of Newtonian fluid [10] , and if just Put $\beta_3 = 0$ equation (1) along with (2) descries the state of second-order:

$$\nabla \boldsymbol{V} = 0, \tag{4}$$

$$\rho(\boldsymbol{V}\nabla\boldsymbol{V}) = \nabla T + (\boldsymbol{J}\times\boldsymbol{B})\ , \tag{5}$$

$$\rho Cp(\mathbf{V}\nabla T) = K\Delta T\ , \tag{6}$$

where the continuity, momentum and energy are represented by equations (4), (5) and (6) respectively. The density is ρ and $(\boldsymbol{J}\times\boldsymbol{B})$ is the Lorenz force vector. The fluid is supposed to be stable and laminar. Substituting the stress tensor T from (1) into (5) yields:

$$\rho(\boldsymbol{V}\nabla V) = -\nabla P + \mu(\nabla^2\boldsymbol{V}) - \sigma u B^2\ . \tag{7}$$

The components of velocity indicated by (*u*, *v*, *w*) are corresponding to *X*, *Y*, *Z* direction respectively [8], to find a compatible solution with the continuity of the formula :

$$u = \frac{Ux}{D}f'(\eta), v = -Uf(\eta), w = \frac{D^2 g\rho}{\mu}h(\eta)\ , \tag{8}$$

where $\eta = y/D$ and the prime denoted the differential with respect to η. The velocity field conditions of the boundary are:

$$f(0) = 0, f(1) = 1, f'(0) = 0, f'(1) = 0, f''(0) = 24, f'''(0) = -48 \tag{9}$$

It follows from (7) and motion equation that:

$$\frac{\partial P}{\partial x} = \frac{Ux}{D^2}[Re(ff'' - (f')^2) + f''' - Mf' + \alpha(-ff'''' + 2f'f''' + 3(f'')^2) + \beta(2(f'')^2)] + \gamma(4f'' + 8f'''') \tag{10}$$

$$\frac{\partial P}{\partial \eta} = -Reff' - \frac{\mu U}{D}\left[f'' + \alpha\left(ff''' + 6f'f'' + \frac{8x^2}{D^2}f''f'''\right) + \beta\left(8f'f'' + \frac{2x^2}{D^2}f''f'''\right) + \gamma(xf'f'' + f'f'')\right], \tag{11}$$

where *Reynolds number* (*Re*), (*M*) is the *Hartmann number*, and *α*, *β* and *γ* are the dimensionless numbers, are defined respectively:

$$Re = \frac{\rho UD}{\mu},\quad M = \frac{\sigma uB}{\mu},\quad \alpha = \frac{U\alpha_1}{\mu D},\quad \beta = \frac{U\alpha_2}{\mu D}\quad and\quad \gamma = \frac{U\beta_3}{\mu D^2}. \tag{12}$$

Integrating (11) with respect to η, we have got the equation:

$$P(x,\eta) = -\frac{1}{2}f^2 + \frac{\mu U}{D}\left[-f' + \alpha\left(ff'' + 3(f')^2 + \frac{4x^2}{D^2}(f'')^2\right) + \beta\left(\frac{x^2}{D^2}(f'')^2 + 4(f')^2\right) + \gamma\left(\frac{x}{2}(f')^2 + (f')^2\right)\right] + G(x) \tag{13}$$

Where $G(x)$ is an arbitrary function. If the equation (13) is a differentiable regarding x yields:

$$\frac{\partial P}{\partial x} = \frac{\mu U}{D}\left[\frac{8x}{D^2}\alpha(f'')^2 + \frac{2x}{D^2}\beta(f'')^2 + \frac{1}{2}\gamma(f')^2\right] + \frac{dG}{dx} \tag{14}$$

Combining (14) and (10)

$$\frac{dG}{dx} = \frac{\mu Ux}{D^3}[Re(ff'' - (f')^2) + f''' - Mf' + \alpha(-ff'''' + 2f'f''' + 3(f'')^2) + \beta(2(f'')^2) + \gamma(4f'' + 8f'''')] - \frac{\mu U}{D}\left[\frac{8x}{D^2}\alpha(f'')^2 + \frac{2x}{D^2}\beta(f'')^2 + \frac{1}{2}\gamma(f')^2\right] \tag{15}$$

And

$$\frac{D^3}{\mu Ux}\frac{dG}{dx} = [Re(ff'' - (f')^2) + f''' - Mf' + \alpha(-ff'''' + 2f'f''' - 3(f'')^2) + \gamma(\tfrac{1}{2}(f')^2 + 4f'' + 8f'''')]\ , \tag{16}$$

The quantity in (16) should be independent of η. Thus, the following equation for *f* is:

$$f''' + Re(ff'' - (f')^2) - Mf' + \alpha(-ff'''' + 2f'f''' - 3(f'')^2) + \gamma(\tfrac{1}{2}f'^2 + 4f'' + 8f'''') = S\ , \tag{17}$$

Where the value of *S* arbitrary constant is:

$$S = f''''(0). \tag{18}$$

Now differentiating (17) with respect to η yields:

$$f''' + Re(ff'' - (f')^2) - Mf' + \alpha(-ff'''' + 2f'f''' - 3(f'')^2) + \gamma(\tfrac{1}{2}(f')^2 + 4f'' + 8f'''')] \tag{19}$$

by using (17), $G(x)$ can be formulated as:

$G(x) = \frac{\mu U x^2}{2D^3} S + S_0,$
(20)
Where C_0 is the integration constant. Adding $G(x)$ from (20) to (13):
$P(x,\eta) = -\frac{1}{2} f^2 + \frac{\mu U}{D}\Big[-Mf' + \alpha\left(ff'' + 3(f')^2 + \frac{4x^2}{D^2}(f'')^2\right) + \beta\left(\frac{x^2}{D^2}(f'')^2 + 4(f')^2\right) + \gamma\left(\frac{x}{2}(f')^2 + (f')^2\right)\Big] + \frac{\mu U x^2}{2D^3} S + S_0.$
(21)
From (21), the variation of pressure can be obtained in a dimensional form in x and y-direction as follows:
$P(x) = \frac{P(0,\eta) - p(x,\eta)}{\rho U^2} = -\frac{1}{Re}(4\alpha(f'')^2 + \beta(f'')^2 + \gamma\frac{1}{2}(f')^2 + \frac{1}{2} f''''(0))(\frac{x}{D})^2$ (22)

$P(y) = \frac{P(x,0) - p(x,\eta)}{\rho U^2} = \frac{f^2}{2} + \frac{1}{Re}(f' + \alpha(ff'' + 3(f')^2) + 4\beta(f')^2 + \gamma(f')^2)$ (23)
The linear operators and initial guesses are chosen as the following:
$T = T_0 + (T_1 - T_0)\theta(\eta).$
(24)
Where T_0 and T_1 are the temperatures of impermeable and plates of porous respectively, with a constant value. Equations (8) and (24) are substituted into (6) and that is lead to the following equation:
$\theta'' + Pef\theta' = 0,$
(25)
Where $Pe = \rho UDcp/k$ is referred to as *Peclet number (Pe)*. Equation (25) is solved depending on these boundary conditions:
$\theta(0) = 0, \theta(1) = 1$
(26)

3. Solution Using HAM:

HAM is used to solve (19) subject to the conditions of the boundary (9).

$f_0(\eta) = 3\eta^2 - 2\eta^3.$
(27)

The initial guess approximation for $f(\eta)$ is:

$L_1(f) = f^v.$
(28)

As the property of the auxiliary linear operator is:

$L(c_1 + c_2\eta + c_3\eta^2 + c_4\eta^3 + c_5\eta^5) = 0$.
(29)

And $c_i (i = 1,2,3,4,5)$ are constants. Let $p \in [0,1]$ and h indicates nonzero ancillary parameters, therefore, the following equations are formed:
$(1-p)L_1(f(\eta;p) - f_0(\eta)] = ph_1N_1[f(\eta;p)]$,
(30)
$f(0;p) = 0, f'(0;p) = 0, f(1;p) = 1, f'(1;p) = 1$.
(31)

$N_1[f(\eta;p)] = f''''(\eta;p) + Re\big(f'''(\eta;p)f(\eta;p) - f'(\eta;p)f''(\eta;p)\big) - Mf''(\eta;p) + \alpha\big(-f(\eta;p)f'''''(\eta;p) + f'(\eta;p)f''''(\eta;p) - 8f''(\eta;p)f'''(\eta;p)\big) = 0$.
(32)
For *p=0* and *p=1*:
$f(\eta;p) = f_0(\eta), f(\eta;1) = f(\eta)$.
(33)
When the value of p increases from 0 to 1 leads $f(\eta;p)$ change from $f_0(\eta)$ to $f(\eta)$. By using Taylor's series and using (33):
$f(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m, \; f_m(\eta) = \frac{1}{m!}\frac{\partial^m(f(\eta;p))}{\partial p^m}$, (34)
$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta),$
(35)
The m -order equations of deformation are:
$L[(f_m(\eta) - X_m f_{m-1}(\eta)] = hR^f{}_m(\eta), m = 1,2, \dots$ (36)
The boundary conditions are:
$f_m(0) = f'_m(0) = f_m(1) = f'_m(1) = 0, f''_m(0) = 6$
(37)
Where $R^f{}_m(\eta) = f''''$+Re$\sum_{i=0}^{m-1}(f_{m-1}f'''_i - f'_{m-1}f''_i) - Mf''_{m-1} + \alpha(\sum_{i=0}^{m-1}(-f_{m-1}f'''''_i + f'_{m-1}f''''_i - 8f''_{m-1}f'''_i)) + \gamma(\frac{1}{2}f_{m-1}'^2 + 4f''_{m-1} + 8f''''_{m-1})]$.
(38)
$X_m = \begin{cases} 0 & m \le 1, \\ 1 & m > 1. \end{cases}$
(39)

To find the solution of third-order deformation, we shall apply the symbolic software MATLAB® up to the first few orders of approximation. We found the solution up to third-order approximation and they are:

F_1=((3*γ*h)/280-(Re*h)/140)*η^8+((Re*h)/35-(3*γ*h)/70)*η^7+((M*h)/60-(Re*h)/20-(8*α*h)/5+ (γ*h)/20)*η^6+((24*α*h)/5-(M*h)/20-(2*γ*h)/5)*η^5-2*η^3+3*η^2

F_2=((3*γ*h)/280-(Re*h)/140)*η^8+((Re*h)/35-(3*γ*h)/70)*η^7+((M*h)/60-(Re*h)/20-(8*α*h)/5+ (γ*h)/20)*η^6+((24*α*h)/5-(M*h)/20-(2*γ*h)/5)*η^5 - 2*η^3+3*η^2

F_3=η^6*((M*h)/60-(Re*h)/20-(8*α*h)/5+(γ*h)/20) -η^5*((M*h)/20-(72*α*h)/5+(2*γ*h)/5+(32*γ^2*h^2)/5+(128*γ^3*h^3)/5+(16*M*γ^2*h^3)/5-(1536*α*γ^2*h^3)/5+(4*M*γ*h^2)/5-(576*α*γ*h^2)/5)+η^11*((193*M^2*α*h^3)/277200-(M^3*h^3)/6652800-(M^2*γ*h^3)/277200-(223*M*Re*α*h^3)/184800+(M*Re*γ*h^3)/11088 +(M*Re*h^2)/46200-0+(M*γ*h^2)/420+(204*α^2*h^2)/35-(311*α*γ*h^2)/560+(3*Re*α*h^2)/112+(5*γ^2*h^2)/56-(17*Re*γ*h^2)/280+(3*γ*h)/140-(Re*h)/70)+3*η^2-2*η^3+η^7*((Re*h)/35-(3*γ*h)/70)-η^8*((Re*h)/140-(3*γ*h)/280)-η^5*((M*h)/20-(24*α*h)/5 (2*γ*h)/5).

4. Converging an equation of temperature:

The equation of temperature in this field as below:

HAM is used to solve equation (25) subject to the boundary conditions (26). The linear operators and initial guesses are chosen in the following:

$\theta(\eta) = \eta$.
(40)

While the initial guess approximation for $\theta(\eta)$ is:

$L_2(\theta) = \theta''$.
(41)

As the auxiliary linear operator has the property:

$L(c_1 + c_2\eta) = 0$.
(42)

And $c_i (i = 1,2)$ are constants. Let $p\in [0,1]$ and h referred to the non-zero ancillary parameter. Therefore, zeroth-order equations of deformation are:

$(1-p)L_1(\theta(\eta;p) - \theta_0(\eta)] = ph_2N_2[\theta(\eta;p)]$, (43)

$\theta(0;p) = 0, \theta(1;p) = 1$,
(44)

$N_2[\theta(\eta;p)] = \theta''(\eta;p) + Pe\big(f(\eta;p)\theta'(\eta;p)\big)=0$,
(45)

for p=0 and p=1:

$\theta(\eta;0) = \theta_0(\eta), \theta(\eta;1) = \theta(\eta)$.
(46)

When p increases from 0 to 1 then $\theta(\eta;p)$ change from $\theta_0(\eta)$ to $\theta(\eta)$. By using Taylor's series and (46):

$\theta(\eta;p) = \theta_0(\eta) + \sum_{m=1}^{\infty}\theta_m(\eta)p^m,\ \theta_m(\eta) = \frac{1}{m!}\frac{\partial^m(\theta(\eta;p))}{\partial p^m}$, (47)

$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty}\theta_m(\eta), m = 1,2,\ldots$
(48)

The m-order equations of deformation:

$L[(\theta_m(\eta) - X_m\theta_{m-1}(\eta)] = hR^{\theta}{}_m(\eta)$.
(49)

The boundary conditions are:

$\theta_m(0) = \theta_m(1) = 0$,
(50)

where $R^{\theta}{}_m(\eta) = \theta''_{m-1} + Pe\sum_{i=0}^{m-1}(f_{m-1}\theta'_i)$,
(51)

$X_m = \begin{cases} 0 & m \le 1, \\ 1 & m > 1. \end{cases}$
(52)

To find the solution of second-order deformation, we shall aplly the symbolic software MATLAB® up to the first few orders of approximation. We found the solution up to third-order approximation and they are:

θ_1 =η - (Pe*h*η^4*(2*η - 5))/20

θ_2 =(Pe*h)/2)-η^5*((Pe*h^2)/10+(Pe*h)/5)+η^11*((M*Pe^2*h^3)/2640-(Pe^2*Re*h^3)/2200-(2*Pe^2*∝*h^3)/55+(Pe^2*γ*h^3)/440)-η^12*((M*Pe^2*h^3)/15840-(Pe^2*Re*h^3)/2464-(Pe^2*∝*h^3)/165+(19*Pe^2*γ*h^3)/36960)-η^8*((Pe^2*h^2)/16-(M*Pe*h^2)/3360+(Pe*Re*h^2)/1120+(Pe*∝*h^2)/35-(Pe*γ*h^2)/1120)-η^10*((M*Pe^2*h^3)/1800-(4*Pe^2*∝*h^3)/75+(Pe^2*γ*h^3)/225+(Pe*Re*h^2)/12600-(Pe*γ*h^2)/8400)+η^7*((Pe^2*h^2)/14-(M*Pe*h^2)/840+(4*Pe*∝*h^2)/35-(Pe*γ*h^2)/105-η^13*((Pe^2*Re*h^3)/7280-(3*Pe^2*γ*h^3)/14560)+η^14*((Pe^2*Re*h^3)/50960-(3*Pe^2*γ*h^3)/101920)-(Pe*h*η^4*(2*η- 5))/20.

5. Results:

We will outline and discuss the effect of dimensionless parameters that govern the momentum and energy equations upon the normal, tangential velocities and temperature of vesicant fluid of third-order in a vertical channel. All results are plotted by MATLAB®. Firstly, the axiality parameter h is set to -0.2 and η = 0 to 3. The following results are observed as mentioned in Table (1) below:

Table (1) represents the results for velocity profile and temperature

	Re=34, M=15, α=15 β=10, γ=30, Pe=2.3				*Re*=70, *M*=25, *α*=20 *β*=15, γ=65.3, *Pe*=9.7			
η	F'''' $10^{8}*$	θ'' $10^{3}*$	Px $10^{17}*$	Py $10^{16}*$	F'''' $10^{9}*$	θ'' $10^{4}*$	*Px* $10^{17}*$	*Py* $10^{18}*$
0	0	0	-0.000	0.000	0	0	-0.000	0
0.2	-0.0043	-0.0002	-0.000	0.000	-0.0025	-0.0001	-0.000	0.000
0.4	-0.0060	-0.0006	-0.000	0.000	-0.0031	-0.0002	-0.000	0.000
0.6	-0.0064	-0.0008	-0.000	0.000	-0.0024	-0.0003	-0.000	0.000
0.8	-0.0067	-0.0005	-0.000	0.000	-0.0005	-0.0003	-0.000	0.000
1	-0.0076	0.0008	-0.000	0.000	0.0026	-0.0005	-0.000	0.000
1.2	-0.081	0.0033	-0.000	0.0001	0.0083	-0.0014	-0.000	0.000
1.4	-0.0045	0.0072	-0.000	0.0003	0.0194	-0.0031	-0.000	0.000
1.6	0.0131	0.0131	-0.000	0.0008	0.0415	-0.0039	-0.0001	0.0001
1.8	0.0669	0.2330	-0.000	0.0013	0.0847	0.0000	-0.0008	0.0001
2	0.2036	0,386	-0.001	0.0031	0.1664	0.0082	-0.0039	0.0001
2.2	0.5120	0.0549	-0.0017	0.0036	0.3146	-0.0055	-0.0166	0.0014
2.4	1.1486	0.416	-0.0128	0.0010	0.5723	0.1485	-0.0630	0.0141
2.6	2.3705	-0.983	-0.719	0.0078	1.0081	-0.7225	-0.2196	0.0834
2.8	4.5756	-0.6086	-0.3324	0.1749	1. 6873	-2.4326	-0.7086	0.3897
3	8.3461	-2.0206	-1.3318	1.4341	2.7354	-6.7409	-2.1236	1.5780

Figure (2) shows the M effect on the normal and the tangential velocity components, M has been given the values of 15, 20 and 25 respectively. The following result is observed: as M increases, the tangential velocity component range is fixed.

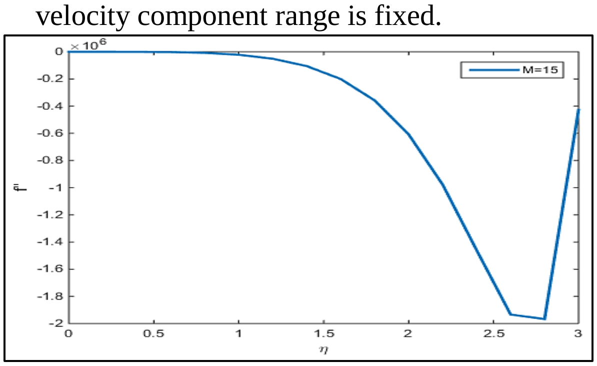

Fig. 2.a Distribution of velocity f for M = 15

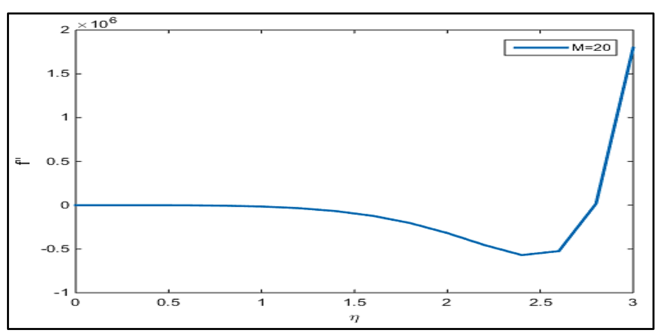

Fig. 2.b Distribution of velocity f for M = 20

The function which corresponds to the components of velocity has been plotted versus η for Re=10, 34 and 70 respectively (see figure (3)). The velocity is increasing according to the increase in Re.

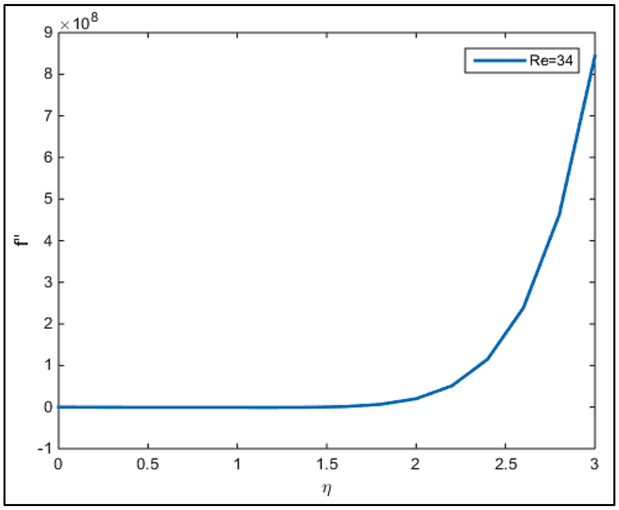

Fig. 3.a Distribution of velocity f for Re=34

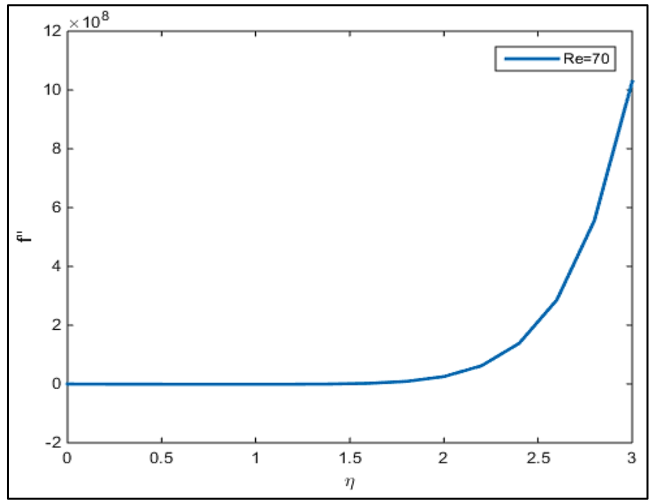

Fig. 3.b Distribution of velocity f for Re=70

Figure(4) illustrates the effect of dimensionless parameter α on the normal velocity profiles for fixed Re=10, M=15 and α=15, 20 and 30 respectively, where the α increasing has a very strong effect on the normal velocity profile.

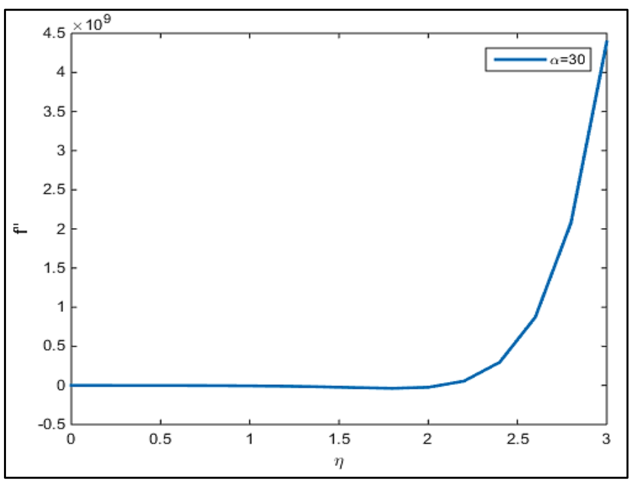

Fig. 4. Distribution of velocity f for α =30

The effect of dimensionless parameter γ on the normal velocity profiles for fixed Re=34, M=15, α = 15 and γ=30, 65.3 and 70 respectively is shown in Figure (5). The increasing of γ has a very strong effect on the normal velocity profile. The velocity will be very high and if the values of $and\ \gamma$ are set to zero then the velocity goes to the state of Newtonian fluid [10].

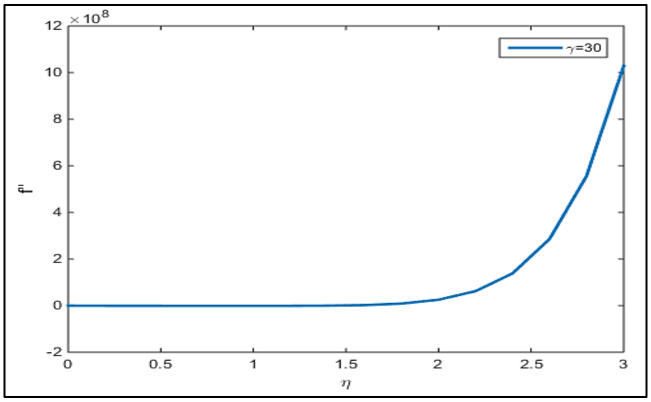

Fig. 5.a Distribution of velocity f for γ=30

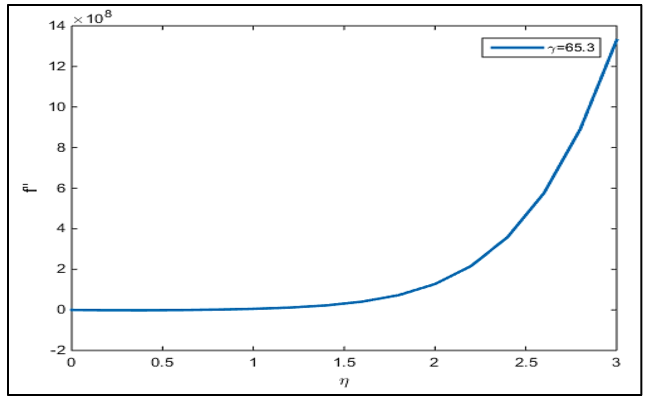

Fig. 5.b Distribution of velocity f for γ=65.3

Figure (6) shows the temperature profiles of sticky fluid in the porous walls vertical channel. When *Re*=34, *M*=15, $\alpha = 20$, $\gamma = 30$ and Pe = 2.3 and 9.7 respectively, the temperature will be decreased.

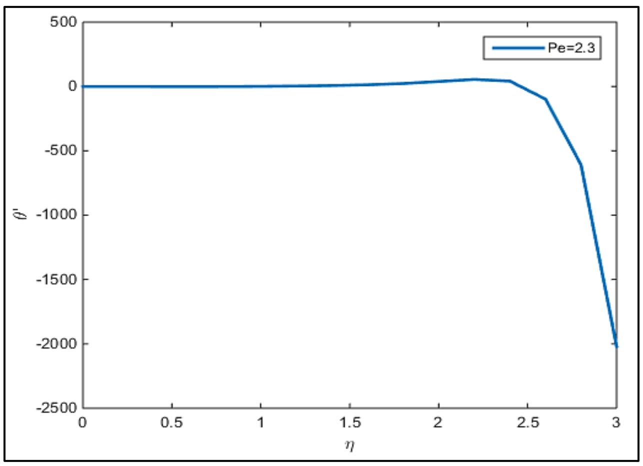

Fig. 6.a Distribution of temperature for *Pe*=2.**3**

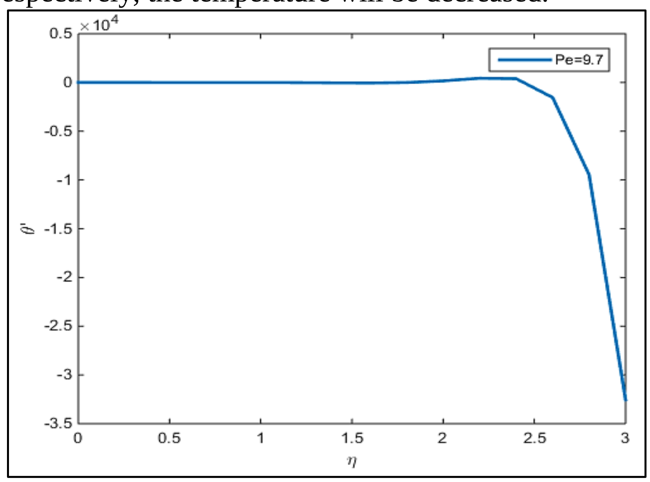

Fig. 6.b Distribution of temperature for *Pe*=9.7

The effect of γ is very strong as shown in Figure (7) because when it's increasing, then the temperature will be very low in the channel.

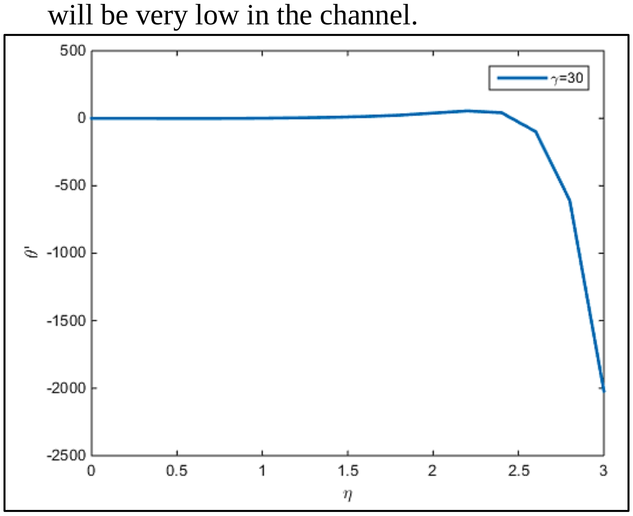

Fig. 7.a Distribution of temperature for $\gamma = 30$

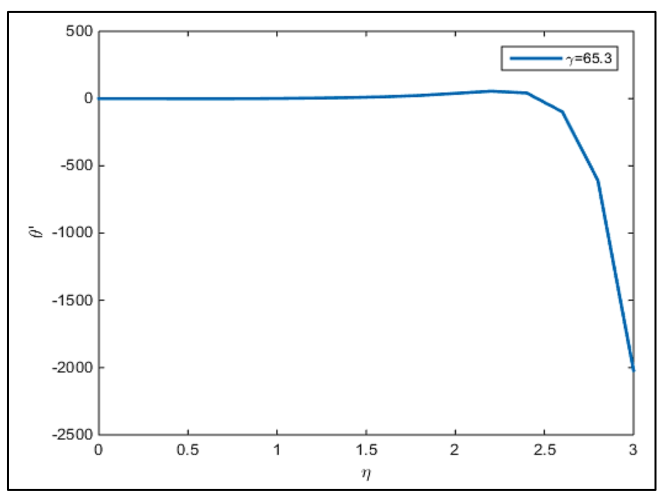

Fig. 7.b Temperature distribution for $\gamma = 65.3$

For the variations of pressure in x and y-directions, when *Re*=10, *M*=15 and γ =30, its obvious that the variations of pressure will increase with increasing of non-Newtonian parameters α, β.

Finally, the variations of pressure (see Figure (8)) will decrease in the x-direction and the increase in y-direction when γ is increasing.

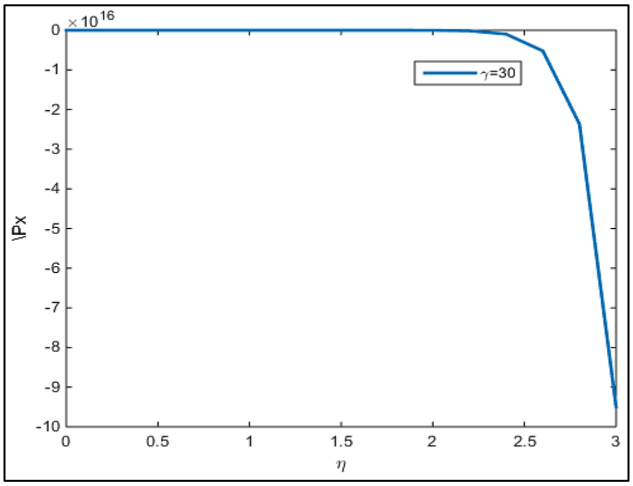

Fig. 8.a Pressure variations in x-direction for $\gamma = 30$

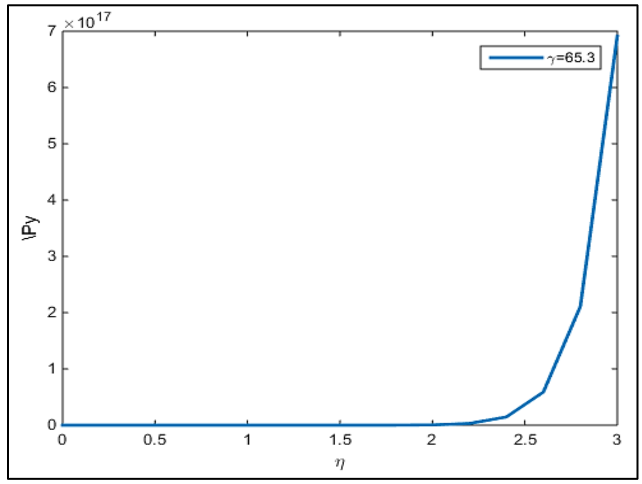

Fig. 8.b Pressure variations in x-direction for $\gamma = 65.3$

6. Conclusions:

An approximate analytical solution of 3rd order non-linear differential equations of fluid under steady flow condition have been studied and discussed. The paper handled this problem which are analytically solved by HAM. According to the obtained results , the following points can be concluded :

- The velocity, pressure and temperature distribution are controlled by non-dimensional parameters (α, β, γ), Re, M, and Pe.
- The effect of β in the velocity is absent, but the effect of α and γ was very strong.
- The existence of γ effect in the heat equation, which is, in turn, will reflex on the effect of Pe leading to channel cooling.
- The influence of γ on the pressure with xy directions is very low, but the increasing value of γ made a low increase of the pressure with y-direction and decreases in the pressure with x-direction.
- The greatest value of the velocity profile and the temperature distribution were obtained when $\eta = 3$.

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