# The effect of noise on digital phase locked loop circuit of second order

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## Abstract:

This paper present a theoretical and experimental work of noise effect on the main performance measure of the second order zero crossing digital phase locked loop (ZDPLL). The loop error probability density function (P.D.F) satisfies the Chapman-kolomogrov equation. From this and the basic equation of approximate expression for the steady state phase error p.d.f. phase error variance and we obtained the loop noise bandwidth on matlab programme. The main measurement used in this paper is value of reliability, phase error variance, probability of correct locked and maximum phase jitter.

**Keyword:** DPLL, PLL, Digital filter, DSP

# تأثير الضوضاء على الدائرة الرقمية المغلقة من الدرجة الثانية

**م. د محمد ابراهيم شجاع**

**الكلية التقنية الهندسية الكهربائية**

الخلاصة:

يتناول البحث تحليل نظري معززا بتصاميم عملية على تأثير الضوضاء على مقايس الاداء الرئيسية لدائرة ضبط الطور الرقمية من الدرجة الثانية عابرة الصفر.ان دالة كثافة الاحتمالية للطور تتبع معادلة جابمان كولومكروف.من هذا ومن المعادلة الاساسية لعمل المنظومة تم اشتقاق معادلات تقريبية (خطية)للطور في حالة الاستقرار معدل الاهتزاز الطوري وعرض حزمة الضوضاء. ان المقايس الرئيسية التي اعتمدت في البحث هي مقدار الاعتمادية معدل الاهتزاز الطوري،احتمالية الاقفال الصحيحة واعلى اهتزاز في الطور.

## List of symbols:

A: single amplitude.

AWGN: Additive white Gaussian noise

C-K: Chapman-kolomogrov equation.

G1, G2, r: digital filter coefficients.

ISN: inverse signal to noise.

NO: spectral density.

P(.):probability of.

P(%): conditional probability density function.

P.D.F.: probability density function.

Q(%): transition probability density function.

ZC-DPLL: zero crossing digital phase locked loop.

DTl: digital tank lock loop

DPLL: digital phase lock loop

DCO: digital control oscillator

Φ: phase error.

$\sigma^{4}$: noise variance

$\theta_o$= phase constant.

W=input frequency

X (t): input signal

A: signal amplitude $w_o=2\pi£_{o.}$

θt: information binary pulse.

N (t): Gaussian additive noise.

## 1. Introduction

The performance of second order ZC-DPLL in the presence of additive white Gaussian noise (AWGN) is presented in this paper. The nature of ZC-DPLL makes the statistical analysis of the phase error process obtained by the studding chapman-kolmogorov (C-K) equation [1] associated with the stochastic difference equation governing the phase error [2].In the analog case, the fokker-planch equation is derived from (C-K) equation associated with the statistic differential equation for the phase error process [1].In the following analysis, module 2π phase error is chosen i.e. by imagining the phase error warp itself around a circle of radius one.

The C-K equation is used to solve the conditional probability density function for markov process. In the process the present value depend only on the last past value, because the phase error is a random variable following markov process will be seen later. C-K equation is applied to the probability density function (p.d.f) of the phase error.

If the signal is band limited with the bandwidth ( Bi) then noise can be approximated by sequence of independent and identically likely Gaussian random variables with zero mean and variance [3].

$\sigma^2 n$=BiN represented the power spectral density over the frequency range of interest.

Equation of analysing and design of second order ZC-DPLL is rewritten here as:

$$\emptyset(k+1) = 2\emptyset(k) - \emptyset(k-1) + k1\sin[\emptyset(k-1)] + k1n(k-1) - r\{k1\sin[\emptyset(k)] + k1n(k)\} \quad \ldots\ldots (1)$$

Where

A: is signal amplitude.

K1=WGiA,$k_n$=W$G_2$A, r=1+$G_2/G_1$, n(k) is the noise component at k instant value ($G_1$, $G_2$) are digital filter coefficient.

The C-K equation which is stated as follows is applied only for markov process.

$$P_{k+1}(\emptyset/\emptyset o) = \int_{-\infty}^{\infty} qk(\emptyset/z)Pk(z/\emptyset o)dz \quad \ldots\ldots (2)$$

Where;

K= time index.

$\emptyset$= $\emptyset o$=initial phase error value.

Pk ($\emptyset/\emptyset o$)=conditional p.d.f of the $\emptyset(k)$ given $\emptyset o$

Qk (./z)=transition p.d.f of $\emptyset(k+1)\, given\, \emptyset(k) = z$

The phase error generated from (2) is non markovian in the present form, however by introducing an auxiliary variable [2].

u(K+1)= (2-1/r)$\emptyset(k+1)-\emptyset(K)+u(k)/r$ ……………………………………………(3)

then (2) can be written as a system of two equations:

$\emptyset(k+1)=-rk1\sin\left[\emptyset(k)+u(k)-rk_n n(k)\right.$…………………………………… (4.1)

u(k+1)=-$\emptyset(k)-(2r-1)k_1\sin[\emptyset(k)]+2u(k)-2r-1)k_n n_k$ ……………………(4.2)

$p_{k+1}\left[\emptyset(k+1)=\emptyset,u(k+1)=\frac{u}{\emptyset_0 u_0}\right]=\int_{-\infty}^{\infty}qk\left[\emptyset(k+1)=\emptyset,u(k+1)=\frac{u}{\emptyset}(k)=\right.$
$\left.x,u(k)=Y\right]P_k\left[\emptyset(k)=X,u(k)=\frac{Y}{\emptyset_{0,}u_0}\right]dXdY$

……………………………….…... (5)

In this form the 2- vector ($\emptyset(k+1),u(k+1)$) is markovian ,hence direct method of C-k equation can be applied [2].

From (4) its obvious that the kth p.d.f is free (independent) of the index k. i.e

q[$\emptyset(k+1)=\emptyset,u(k+1)=\frac{u}{\emptyset}(k)X,u(k),u(k)=Y)$

=q [u(k+1)=u/$\emptyset(k+1)=\emptyset,\emptyset(k)=X,u(k)=Y]$.

q[$\emptyset(k+1)\frac{\emptyset}{\emptyset}(k)=X,u(k)=Y$

=$\delta\left\{u-\left[\left(2-\frac{1}{r}\right)\emptyset-X+\frac{Y}{r}\right]\right\}$ ...…………………………………………………..….. (6)

$$\frac{1}{\pi r\sqrt{2}}\int_{-\infty}^{\infty}\exp\left[\frac{(2\sigma 2-u-X+k_1\sin X)2}{2\delta 2}\right. \quad \dots\dots\dots\dots\dots\dots\dots\dots(7)$$

Pu [$\emptyset(k)=X,u(k)=(2r-1)\emptyset+ru+rx]dx$

Where the conditioning on Øo and Uo has been dropped for simplicity in notation.

After taking expectation and letting k………>∞ in (4)

E [sinØ] $=\emptyset,E[\emptyset]=E[u]$ ………………………………………………………………..(8)

Where E[…] representing expectation squaring and cross multiplying (4) taking expectation and finally letting k…..>∞,the following system of equation are obtained.

E[$\emptyset^2]=E[(-rk_1\sin\emptyset+u)2]+r^2k_n2\sigma n^2$…………………………………….……(9.1)

$E[u^2] = E[(-\emptyset-(2r-1)k_1 sin\emptyset+2u)2+(2r-1)2k_n 2\sigma n2$.....................(9.2)

$E[\emptyset u] = E[(-rk_1 sin\emptyset+u)(-\emptyset-(2r-1)k_1 sin\emptyset+2u)]$+r(2r-1)$k_n 2\sigma_n 2$......... (9.3)

When the DPLL is in the tracking mode and the phase error is small with high probability (high SNR case) then sin$\emptyset \approx \emptyset$.if the approximation is used therefore.

$$E[\emptyset^2] = \frac{(r^2+1)(2-k_1)-2r(2-rk_1)}{c} k_n 2\sigma_n 2$$

$$E[u^2] = \frac{(5r^2-4r+1)(2-k_1)-2r(2r-1)(2-rk_1)}{c} k_n 2\sigma_n 2$$

$$E[\emptyset u] = \frac{2r^2(2-k_1)-(r^2+2r-1)(2-rk_1)}{c} k_n 2\sigma_n 2 \qquad (10)$$

Where C=$k_1[2-k_1)2-(2-2k_1)^2]$

Using (10) for the steady state carapace $\sigma^2 = E(\emptyset^2).then$

$$\sigma^2 = \frac{1}{k_1 2}[\frac{2}{\frac{4}{r+1}-k_1} - 1]k_n 2\sigma_n 2 \qquad (11)$$

The linear loop noise bandwidth W after simplifying (11) is given by

$$\frac{Wl}{Bi} = \frac{2}{\frac{4}{r+1}-k_1} - 1 \qquad (12)$$

Equation (11) can be further simplified to

$$\sigma^2 = \left[\frac{2}{\frac{4}{r+1}-k_1} - 1\right]\frac{\sigma n^2}{A^2} \qquad (13)$$

Thus p.d.f can be re written as:

$$\text{p.d.f. }(\emptyset) = P(\emptyset) = \frac{1}{\sigma\sqrt{2\pi}} \int \exp\left(\frac{-\emptyset^2}{2\sigma^2}\right) d\emptyset \qquad (14)$$

the optimum value of the filter parameters $k_i$,r minimum 4/(rH).At the same time $k_i$ and r must also satisfy the constraints 0<k,4/(rH).its difficult to minimize $\sigma 2$ by differentiations . however the digital computer can be easily programmed to search for the minimum mean square error as both parameters are varied within the mentioned constraints [4].

## 2. Zero crossing DPLL

DPLL type receives sinusoidal signal and sample an input of one to nearby zero .That's why it's called zero-crossing (ZC-DPLL).
There are two kind of (ZC-DPLL), one is (Z1-DPLL)which works on positive-going crossing, and the (ZC2-DPLL) which works on both positive and negative.
(Z1-DPLL) is the more important and simplest for implementation where its interpretation are ineffectual in general behaviour of any DPLL [5 ],hence (ZC2-DPLL) is faster but complicated.
(Z1-DPLL) has been developed in works [ 3,6 ], it present numerical solution for (Chapman-Kolomogorov equation) [7].DPLL was improved in 1982 by advent of digital (tank lock loop) DTL [1 ], where (Z1-DPLL) based on phase detection.

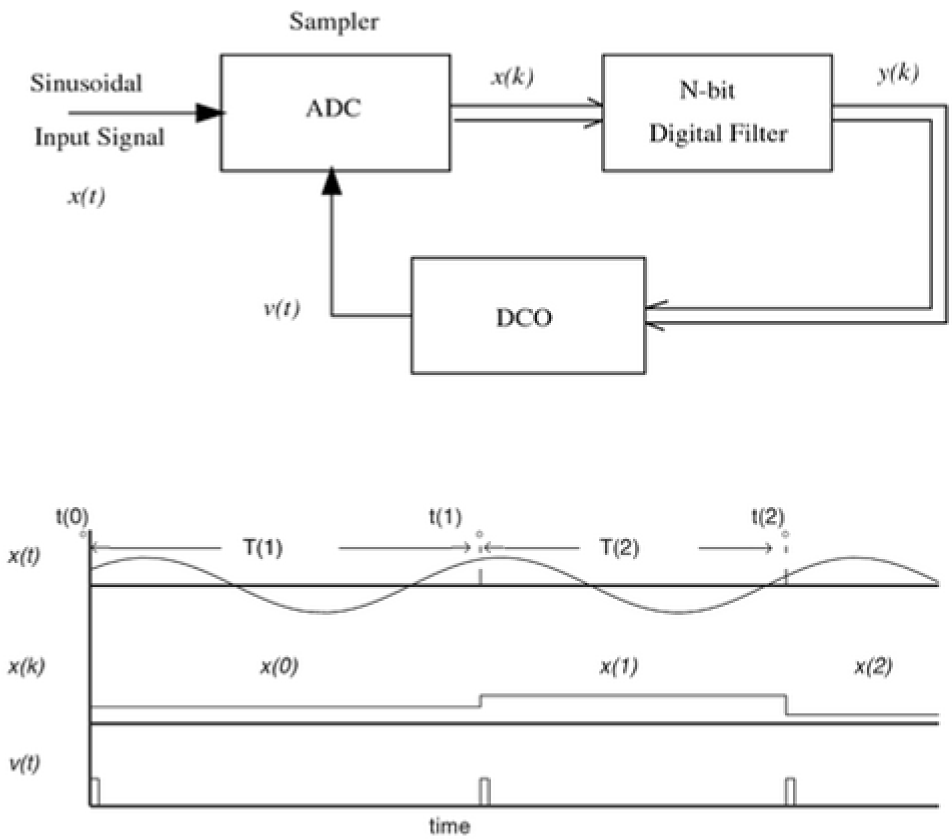



Fig 1: (ZC1-DPLL) sinusoidal with associated waveform

## 3. Digital Controlled Oscillation (DCO)

(DCO) consist of a programmable counter, binary subtract and zero detector.
Subtraction is done by using second complement and full adder, while counter is decreased by only one of each clock. When it reach zero, counter gives pulse on output.
Where it loads counter with binary number ( M-K), ( M-constant, K-input)
M-declare (DCO) free-running frequency $£_0$ where K=0 as:

$$£_{0}= £_c/m \text{.....................(15)}$$

$£_{c}$= counter frequency clock
Period time between $(K-1)^{th}$ and $K^{th}$ obtained by:

$$T(K)=(M-K)\, T_c \text{..........................(15.1)}$$

$T_c= 1/£_c$

Pulse equation can be obtained by :

$$X(t) =A \sin \{w_o t+\theta(t)+n(t) \text{...........(15.2)}$$

X(t) : input signal
A: signal amplitude $w_o=2\pi £_{o.}$
θt: information binary pulse.
N(t) : Gaussian additive noise.

Frequency steps input: $\theta(t)=(w-w_o)\, t+\theta_o$......................................(15.3)

$\theta_o$= phase constant.
W=input frequency (first order loop.)

## 4. System Design and proposed methods:

The experimental system suggested in the work [1] is used to study the behaviour of the loop within the noise. A up system is used to store the phase error sample in its memory, and used later to plot the p.d.f. of the phase error simulated by matlab programming .the Gaussian random variants ($n_k$) are generated in three steps:

1. Generating a pair of random values $X_{1,}$ X2 uniformly distributed over (0, 1) [1].
2. Generating the pair:
   $Y_{1=}$ -21n$x_1$cos $(2\pi x_2)$
   $Y_{2=}$ -21n$x_1$sin $(2\pi x_2)$

   Which are independent and Gaussian distributed with zero mean and unit variance [1].

3. Scaling the variance to the appropriate value according to the S/N value. The probability density function (p.d.f.) is determined by the following equation [1].
   Where,
   N(X): Number of phase error value which falls is range x±w.
   N: Total numbers of phase error.
   W: Narrow interval centred at x.

   Thus p.d.f. is obtained and gain by dividing the full range of (x) into an approximate number of equal width class interval tabulating the number of data value in each class interval and dividing by the product of class interval W and sample size N.

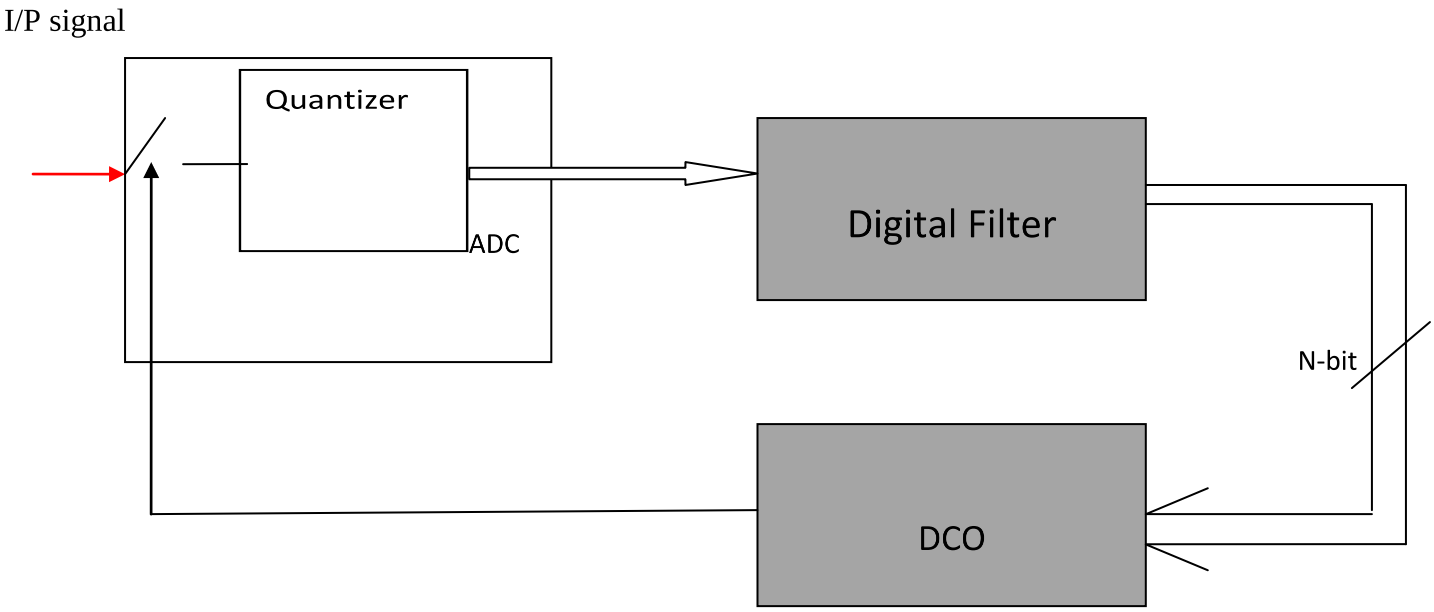



**Figure (2): Block diagram of ZC_DPLL.**

DCO :Digital Controller Oscillator

ADC: Analogue to Digital Converter

## 5. Results:

The main performance measures of the ZC-DPLL in the presence of the noise in the phase error cycle slipping. The p.d.f. of the phase error gives an indication about the phase error slipping. A White additive gaussian noise whose variance is equal to $\sigma^2$ is used in the following matlab simulation results. The actual phase error value are collected from the total simulation points and can be explained in what followes.3000 phase error points are used in the simulation with every tenth value states as the steady states phase error are tabulated and their p.d.f. is plotted. Noise strength is measured by 1/(ISN) which is equal to $\sigma_n^2/A^2$ where A is signal amplitude's/N is related to ISN as 10 LOG (1/OSN).

Fig (3) shows the p.d.f. of the phase error of $k_1$=1,r=2 and S/N=10dB (ISN=0.1) with the linear approximation that is derived earlier .A close matching is noticed between the simulation and the linear approximation. The figure is repeated for a positive frequency of set and plotted in fig (4). The mean value of phase error in this figure is expected when the second order ZC-DPLL is subjected to frequency step input .Fig (5) shows the p.d.f. of the phase error when a negative frequency offset is applied to the loop. The loop performance with noise is improved by decreasing the value of (r) as shown in Fig (6a).This is slightly affected by the $k_1$ value as shown in Fig (6b).

The effected of (r) on noise performance is shown practically with S/N=8dB and r=1.2 with $G_1$=0.40H (hexadecimal) and $G_2$=0.20H,and p.d.f. of the phase error is shown in Fig(6a).With the same S/N ratio and r=2 and $G_1$=0.40H,the p.d.f. of phase error is drawn in Fig (6b).The above two figures shows that the decrease in value of (r) increases the performance of loop. i.e. low phase error as shown in Fig(9).( Jitter and higher probability of locking). The main performance measures of the loop in the presence of noise are reliability phase error standards deviation, probability of correct looking and maximum phase jitter.

The reliability is defined by the equation reliability %= (total time-slip time) /total x100%. The probability of correct locking is probability that the loop locks at correct phase. Maximum phase jitter is the maximum spread of p.d.f. curve and represents measures of loop tendency to cycle slip. The reliability of ZC-DPLL vs input S/N , standard deviation vs S/N probability correct locked vs S/N ,and maximum phase jitter vs S/N are Plotted versus input signals /noise and shown in figures (8,9,10,11) it can be noticed that the performance of loop degraded as the S/N decreases.

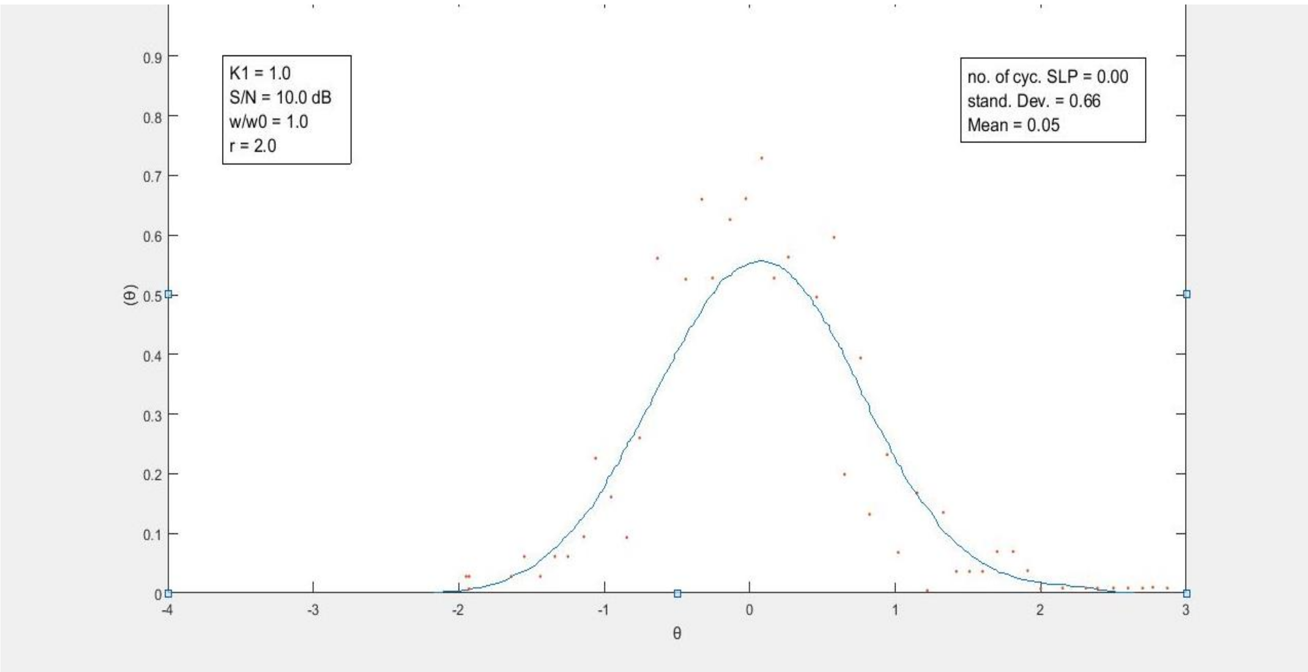



**Figure (3):** Steady state p.d.f for second order ZC-DPLL

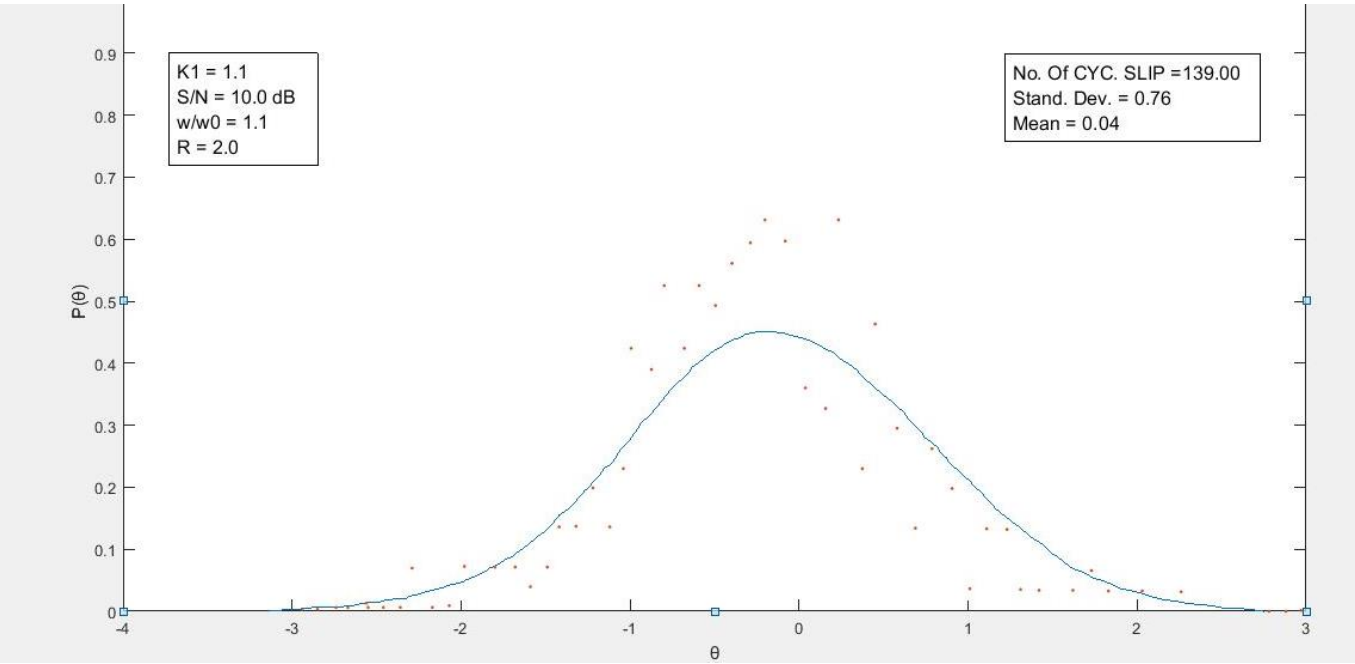



Figure (4): Steady state p.d.f for second order ZC-DPLL

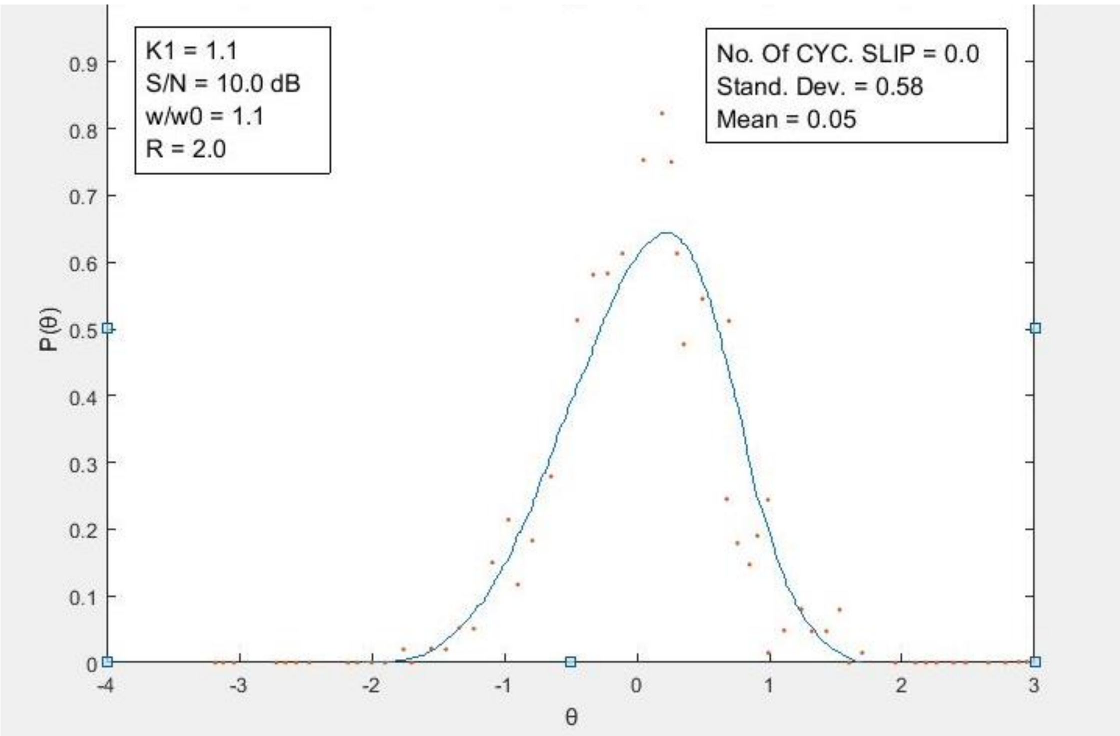
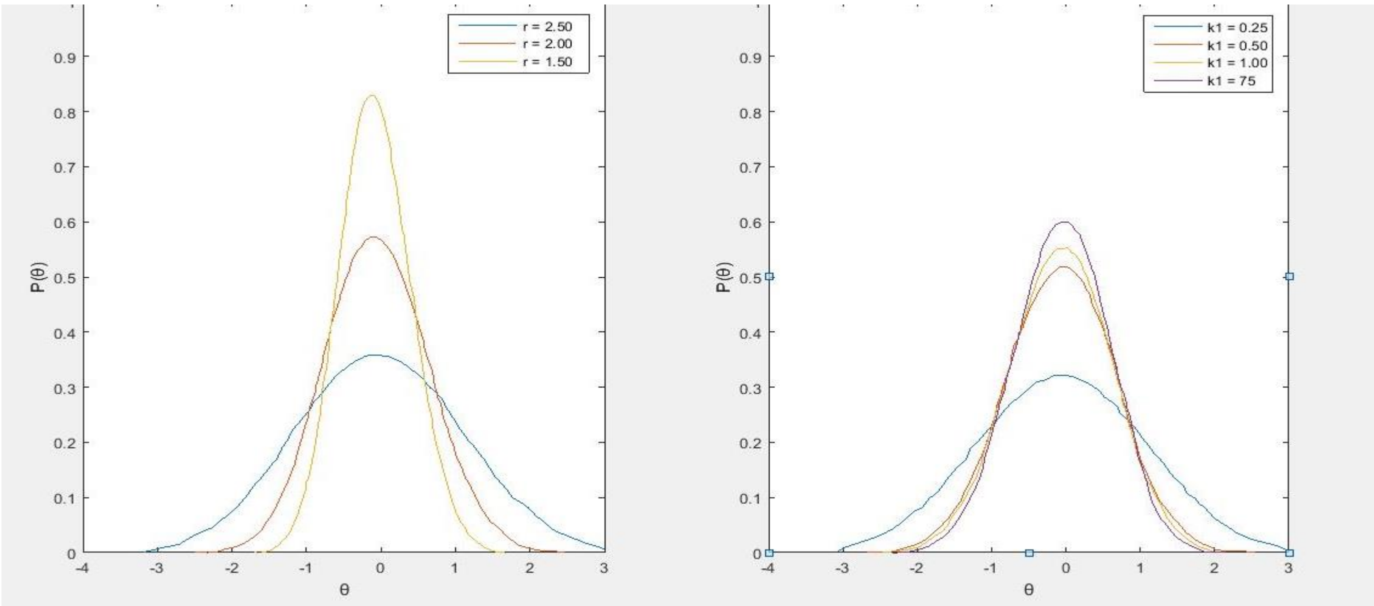



**Figure (5)**: Steady state p.d.f for second order ZC-DPLL

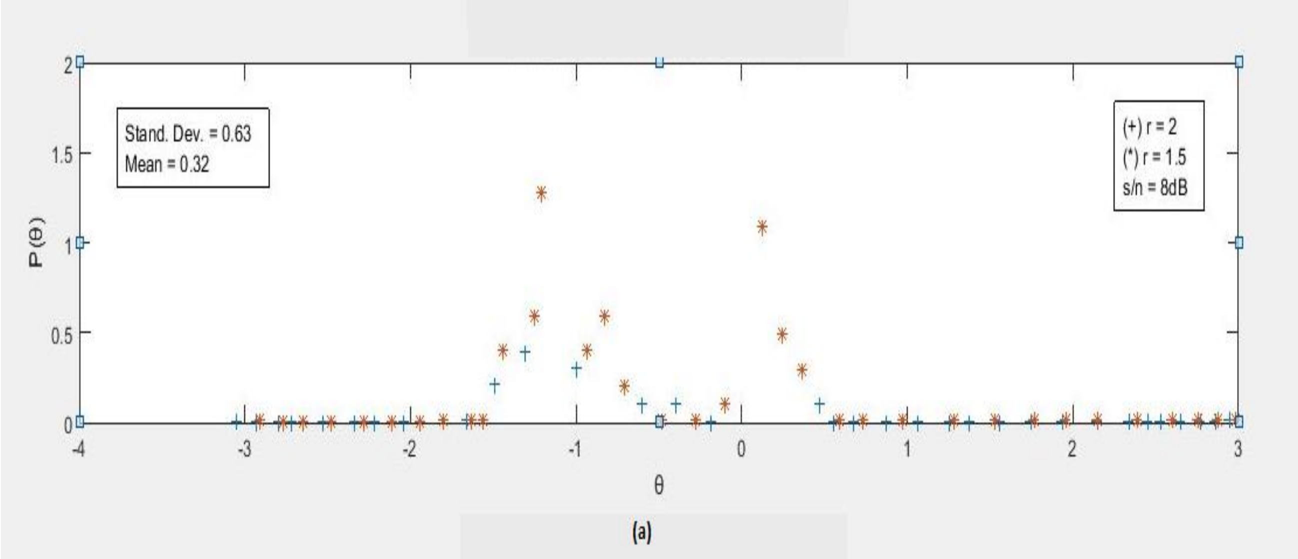



(a) (b)

**Figure (6):** steady state p.d.f for second order ZC-DPLL

(a) With various k value

(b) With r value



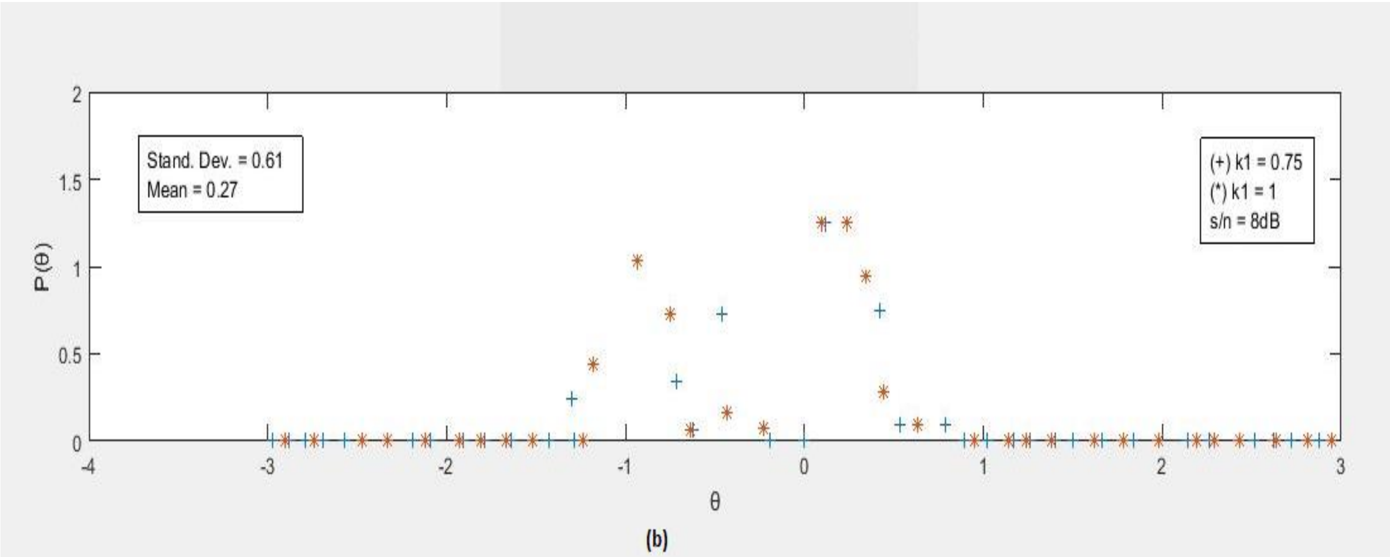




Figure (7): p.d.f for experimental second order ZC-DPLL

(a) With various r value and S/N=8dB

(b) With various K1 value and S/N=8dB

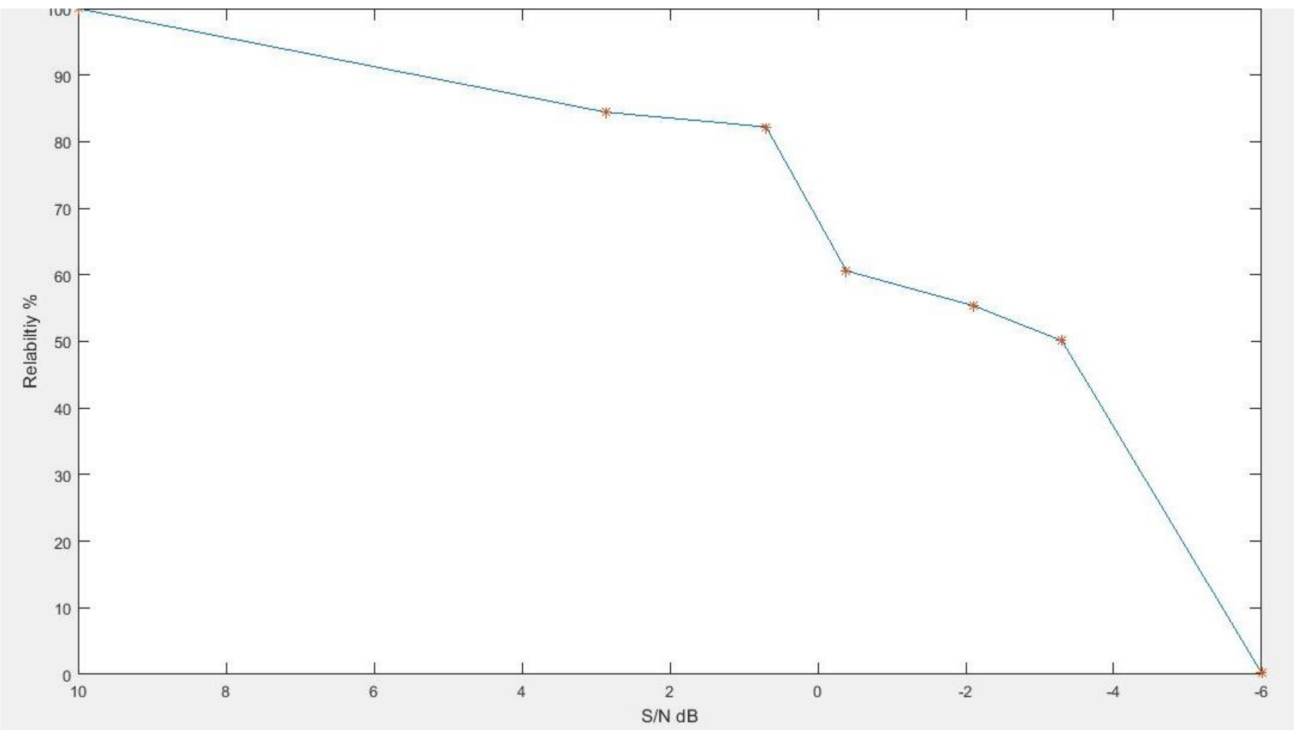



Figure (8): Reliability of second order ZC-DPLL vs. Input S/N

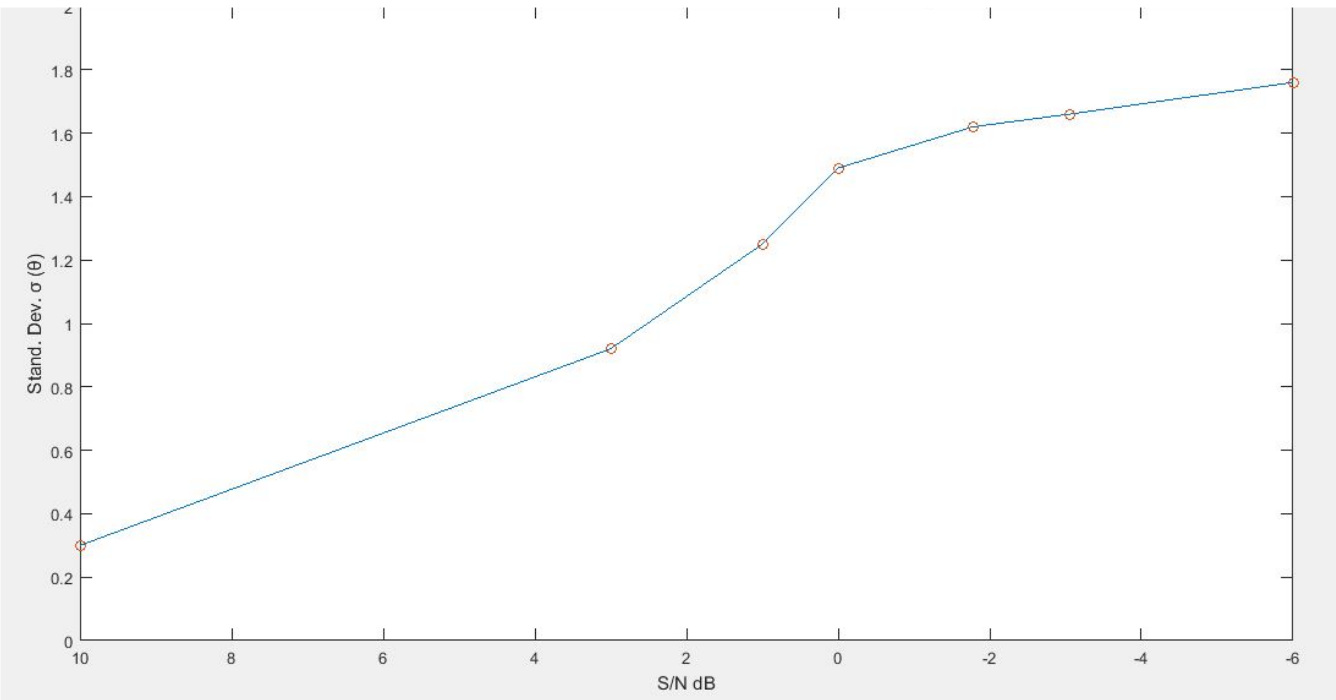



Figure (9): Phase error standard deviation of second order ZC-DPLL vs. input S/N

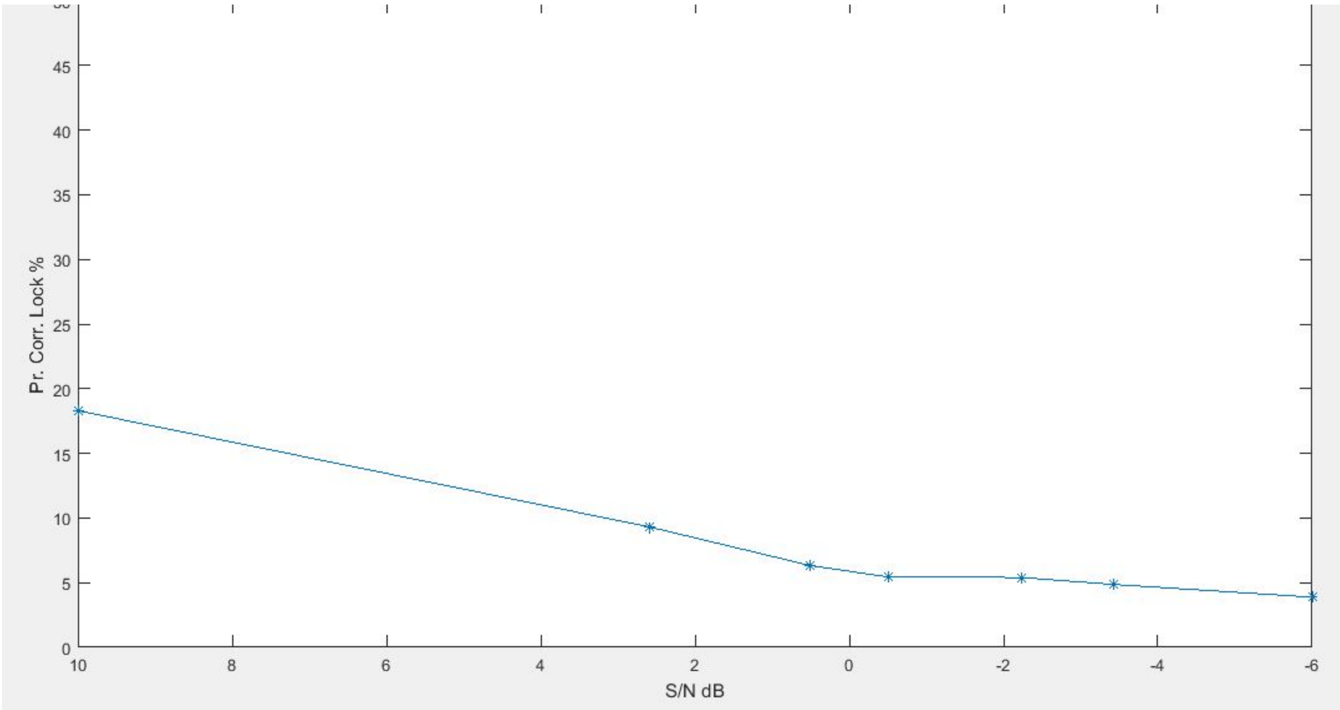



**Figure (10):** Probability of correct locking of second order ZC-DPLL vs. input S/N

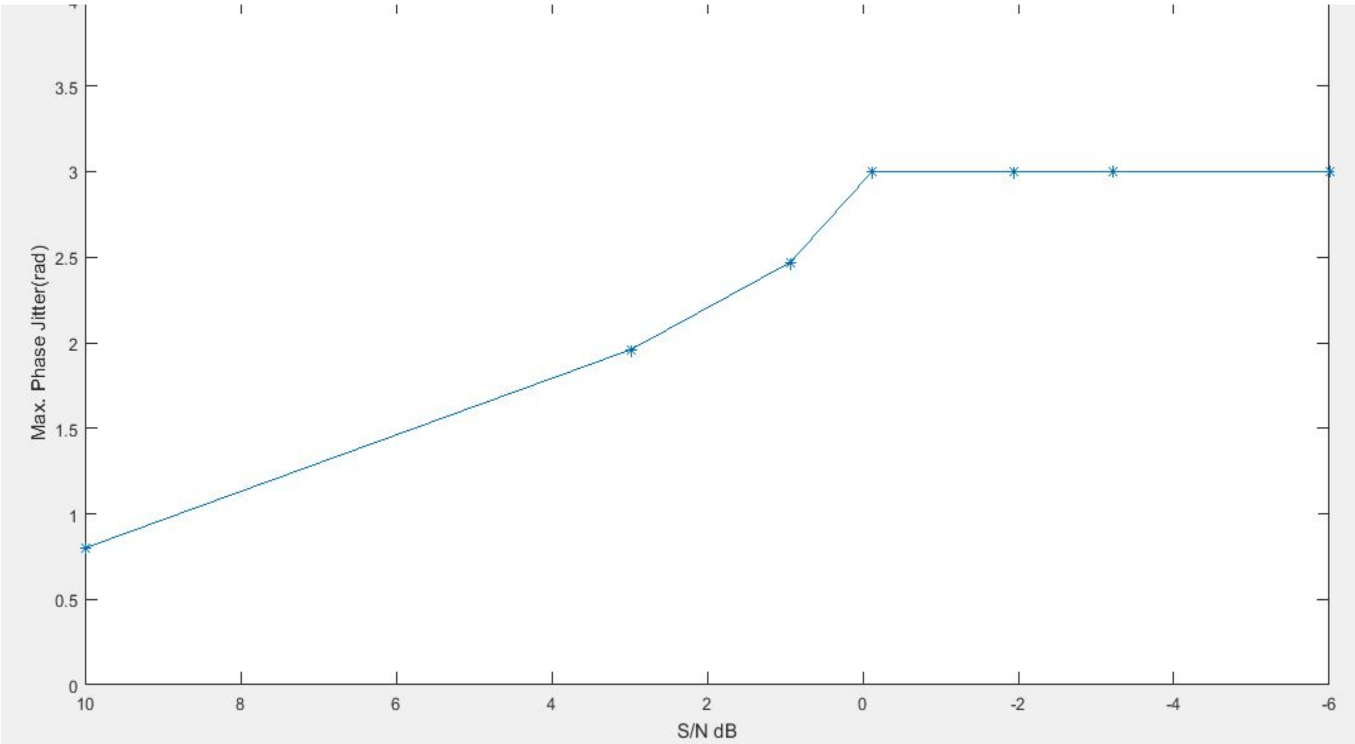



**Figure (11):** Phase error jitter of second order ZC-DPLL vs. input S/N

## 6. Conclusions:

A second order zero crossing digital phase locked loop(ZC-DPLL) is analysed in the presence of the effect noise. An approximate expression for the steady state phase error probability density function phase error variance and noise effect loop bandwidth are obtained and given by equation (14, 13, and 12).The loop noise performance is affected by the loop filter gain $G_1$, G2 .

## 7. Reference:

[1] kasim M. hamza and saleh R.AL.Araji,"analysis and design of the second order zero crossing Digital Phase locked loop in the absence of noise." J. Electronic.& computer. Res vol.1(2) /P 49-70.Electronic and computer scientific research council-Baghdad. Oct, 1987

[2] Weinberg A.and Liu B. "Discreste Time Analysis Of Non-uniform sampling first and second order-Order Digital Phase Locked Loop",IEEE Trans Gommon.Technol.Vol COM22,pp123-137.feb 1974.

[3] Q. Nasir and S. R. Al-Araji, "Optimum Performance Zero Crossing Digital Phase Locked Loop using Multi-Sampling Technique," IEEE International Conference on Electronics Circuits and Systems, Sharjah, pp. 719-722. 14-17 December 2003,

[4] Schwartz M. "information Transmission Modulation and noise"Mc-Graw Hill,pp357. 1980

[5] Gill G.S and Gupya S.C."on higher – order disrcreste digital locked loop"IEEE trans AES-8.pp615-623.1972

[6] M. Nandi, 'Optimization of zero crossing digital phase locked loop performance in carrier synchronization system', Int. Journal of Electronics and Communication. Eng., vol. 9, pp.77-85, no.1, 2016

[7] Q. Nasir, "Chaos Controlled ZCDPLL for Carrier Recovery in Noisy Channels," Wireless Personal Communications, Vol. 43, pp. 1577-1582. No. 4, December 2007