

Design of Digital Filters for Analysis of EEG Signal

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Abstract

Analytical sequence technique was designed and applied on normal and epileptically human electroencephalography (EEG) data. This technique was composed of three stages. First extraction of EEG spikes and rejection noise, slow and artifacts components. Second determining amplitude threshold which describe spike incidence. Third representation of spikes per second on a bar chart. A set of a band pass filter was designed for extraction of EEG spikes. An accurate detection of spikes was obtained with band pass digital filters of double zero at ($z=\pm 1$), single pole placed on a circle radius (r_1 and r_2) and 4th order pole was placed at the origin. A threshold program was successfully used to recognize the spikes incidence. Bar chart program was carefully used to count the number of incidence spikes per second on EEG data.

Kew words: Digital Filters, Signal Analysis, Digital Signal Processing

تصميم مرشحات رقمية لتحليل إشارة تخطيط الدماغ

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المستخلص

في هذه الدراسة تم تصميم وتطبيق تقنية لتحليل إشارة تخطيط الدماغ لأشخاص غير مصابين وآخرين مصابين بمرض الصرع لكلا الجنسين. تتكون هذه التقنية من ثلاثة مراحل: تم في المرحلة الأولى استبعاد النتوء وإزالة الضوضاء، الإشارة الضعيفة والتداخلات الموجودة في الإشارة. أما في المرحلة الثانية فقد تم تحديد حد العتبة للسعة لوصف حدوث النتوء. وفي المرحلة الثالثة تم تمثيل عدد النتوءات لكل ثانية بواسطة الأعمدة المستطيلة. تم تصميم عدد من المرشحات الرقمية التي تسمح بمرور حزمة معينة من الإشارة وذلك لاستبعاد النتوء من إشارة تخطيط الدماغ. وقد وجد بأن أدق كشف للنتوء كان باستخدام المرشح الرقمي الذي يتكون من صفر مزدوج في ($Z=\pm 1$) وقطب مفرد موضوع في دائرة نصف قطرها (نق 1 نق 2) وأربعة أقطاب موضوعة في مركز الدائرة. عند تطبيق برنامج حد العتبة، تم الكشف عن النتوءات بنجاح. ولدى تطبيق برنامج الأعمدة المستطيلة لحساب عدد النتوءات لكل ثانية، استطعنا من خلاله تشخيص الحالة المرضية للمريض.

كلمات المفتاح: الفلاتر الرقمية، تحليل الإشارة، معالجة الإشارة الرقمية.

Introduction

The analytical sequence technique which is used in this study based on digital signal processing (DSP) for EEG analysis, which is help the EEGer indiscrimination between abnormal spikes and artifacts and normal.

The designed analytical technique involves three stages: first stage was deal with extraction of spikes from slow wave, noise and artifacts. These tasks can be achieved by development of a set of z- transform digital filters. Second stage was for development of threshold routine which recognized point event sequence. Third stage was for

developed bar chart program to illustrated number of incidence spikes per second on EEG data.

Theory

Digital Filter

Digital filters are a very important part of (DSP). A digital filter is just a filter that operates on digital signals, such as sound represented inside a computer. It is a computation which takes one sequence of numbers (the input signal) and produces a new sequence of numbers (the filtered output signal)[1].



Figure (1): A block diagram of a basic digital filter[2]

Transfer Function

The input x_n and output y_n sequences of a digital filter both represent signals sampled at discrete, uniformly spaced, time increments t_n . A finite impulse response (FIR) digital filter takes $N+1$ of the most recent samples of x_n , multiplies them by $N+1$ coefficient, and sums the result to form y_n . For an infinite impulse response (IIR) filter, the M previous samples of y_n are weighted and added in as well. In other words, an IIR filter uses feedback.

This is expressed mathematically by [3]:

$$Y_n = \sum_{k=0}^N b_k x_{n-k} + \sum_{k=1}^M a_k y_{n-k} \quad \text{-----(1)}$$

Z-transform

The z-transform is used to describe the properties of a sampled data signal or system, and it provides useful methods of representing the sampled data signal or system by either a finite set of poles and zeros(frequency-domain representation) or by a linear difference equation (time - domain representation) [4].

Just as analog filters are designed using the Laplace transform, recursive digital filters are developed with a parallel technique called the z-transform. To reinforce that the Laplace and z-transforms are parallel techniques, we will start with the Laplace transform and show how it can be changed into the z-transform. The Laplace transform is defined by the relationship between the time domain and s-domain signals:

$$X(s) = \int_{t=-\infty}^{\infty} x(t)e^{-st} dt \quad \text{-----}(2)$$

where $x(t)$ and $X(s)$ are the time domain and s-domain representation of the signal, respectively.

The Laplace transform can be changed into the z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad \text{-----}(3)$$

Eq.(3) represents the standard form of the z-transform, which defines the relationship between the time domain signal, $x[n]$, and the z-domain signal, $X(z)$ [5].

The Digital filter transfer function

The transfer function of the filter is the ratio $Y(z)/X(z)$, where $Y(z)$ and $X(z)$ are the z-transforms of the output and input signals respectively.

$$H(z) = \frac{Y(z)}{X(z)} \quad \text{-----} (4)$$

Multiplying the input transform $X(z)$ by the transfer function $H(z)$ gives the output transform $Y(z)$ [4].

Transfer function in pole –zero form

An important feature of the z-domain is that the transfer function can be expressed as **poles** and **zeros**. This provides the second general form of the z-domain:

$$H(z) = \frac{(z - z_1)(z - z_2)(z - z_3) \text{-----}}{(z - p_1)(z - p_2)(z - p_3) \text{-----}} \quad \text{-----} (5)$$

Each of the poles (p_1, p_2, p_3, \dots) and zeros (z_1, z_2, z_3, \dots) is a complex number[5].

The behavior of the digital filter is governed by the location of its poles and zeroes in the z-plane [5]. One of the most important characteristics of the z-plane is that the region of filter stability is mapped to the inside of the unit circle on the z-plane. Given the $H(z)$ transfer function of a digital filter, the poles location of this function determine stability of the filter. If all poles are located inside the unit circle, the filter will be stable. On the other hand, if any pole is located outside the unit circle the filter will be unstable [6]. As shown in figure (2).

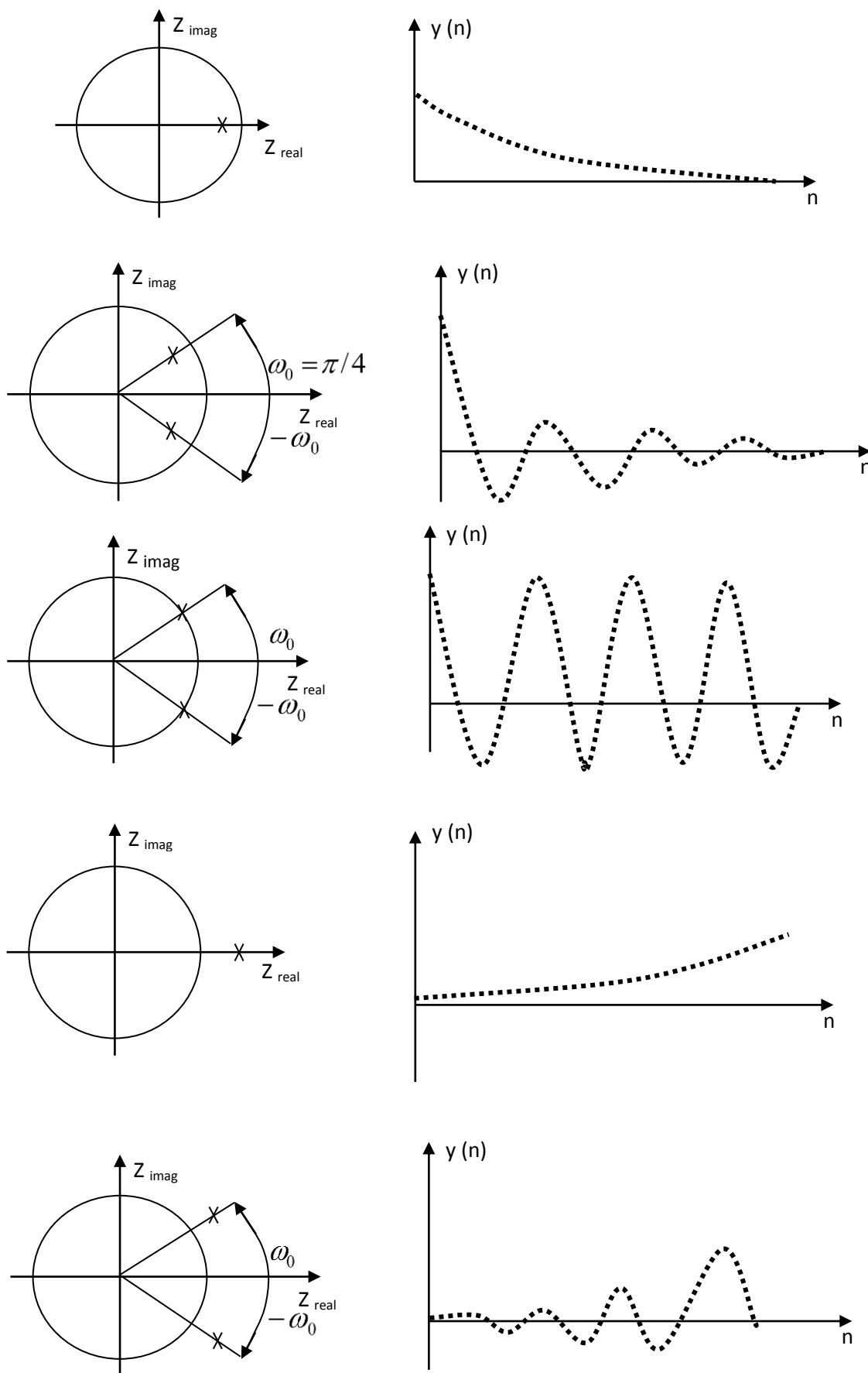


Figure (2): Various $H(z)$ pole location and their discrete time-domain [6]

Results

A- Data plotting

EEG recordings were plotted on computer by using program MATLAB, so that the computer analysis could be compared with the original signal .In EEG analysis ,the sampling rate was chosen to be (160 Hz) to avoiding loss of information .The recorded data was displayed on the computer and compared to actual data, no loss of information was noticed , this was confirmed by medical specialist.

A computer printout of 16 channels of EEG recordings are shown in figure (3a) & figure (3b) respectively with **(40000-60000)** points of data on each channel, each channel has been analyzed alone .In figures (3a)and (3b) the signal from the beginning to $(8.5 \cdot 10^4)$ points and the last $(0.2 \cdot 10^4)$ points represent the calibration of EEG .Calibration is selected simultaneously for all channels by a switch on the control unite .A calibration signal of $(100\mu V, 0.5Hz)$ square wave was recorded at the beginning of each record , which was used as a scaling signal of the actual absolute amplitude values.

To find the number of points per second we divided the total number of points over the total measuring time ,then we plot the EEG signal for one minute ,which consist of ten segments ,each segment represent 6 sec

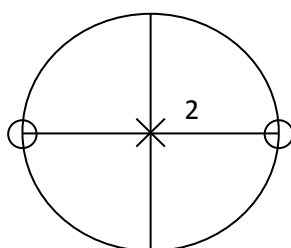
as shown in figure (4). This figure shows a computer print out of ten tracing (6 sec long)of one minute of data of channel two ,channel two was chosen for the processing the signal which has lowest noise contamination .On each trace the **1250** points correspond to **(6)**seconds of data.

B-Analytical Technique

In this work, digital filtering operations form an important part of EEG signal analysis procedures. The signal to be analyzed consists of two main components, both of interest to diagnosis epilepsy cases and to the physiologist. These are action potential "spikes" and slow rhythmic wave. The tasks required a suitable filter for detecting and separating the spike from the slow wave and for noise reduction .For such operations, different configuration of band pass digital filters have been designed using z-transform techniques.

The band pass filters were designed to extract spikes and reject **(80Hz)** on data sampled at **(160Hz)**.

Firstly, a very simple band pass filter was designed as shown in figure (5), this filter has a single zero at **(z = ±1)** to get a rejection of low frequencies and noise components .The second order poles were placed at origin for phase response.



The

Figure (5): Pole-Zero Configuration of Simple Band Pass Digital Filter with single zero at $(z = \pm 1)$

To find recurrence formula:

Since $H(z) = \frac{Y(z)}{X(z)}$ ----- (7)

$H(z) = (1 - z^{-2}) = \frac{Y(z)}{X(z)}$ -----(8)

$Y(z) = X(z) - z^{-2}X(z)$

In general, we may transform a term such $a_1 z^m \cdot X(z)$ into $a_1 \bullet X(n+m)$ or a term $a_2 z^k \bullet Y(z)$ into a term $a_2 \bullet y(n+k)$ where m and k are integers. The reason of this is that z may be thought of as a shift operator multiplication by z is equivalent to time shift of T seconds [2]. Then the recurrence formula becomes

$\therefore y(n) = x(n) - x(n-2)$ ----- (9)

When this band pass filter was applied on the EEG signal low frequencies were extracted as shown in figure (6).

A second band pass digital filter was designed as shown in figure (7), which had a double zero at (z=1) to achieve a complete rejection of zero frequency, and a single zero at (z=-1) to reject noise.

The complex conjugate zero-pair were placed on the unit circle at ($z = e^{\pm j\theta}$) with ($\theta = \pm 45^\circ$) to extract frequency of (8-13) Hz and. (36-44)Hz. In addition a 5th order pole was placed at origin to maintain the condition so that the filter output is calculated from previous output and nth earlier inputs.

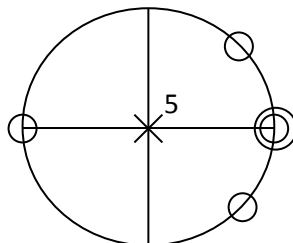


Figure (7): Pole-Zero Configuration of Band Pass Digital Filter with a double zero at (z=1), single zero at (z=-1), complex conjugate zero-pair at ($z = e^{\pm j\theta}$) with ($\theta = \pm 45^\circ$) and a 5th order pole at origin.

The z-transform function of this filter was found to be

$H(z) = \frac{(z-1)(z-1)(z+1)(z-e^{-j\theta})(z-e^{+j\theta})}{z^5}$ ----- (10)

$H(z) = \frac{(z^2-1)(z-1)(z-\cos\theta+j\sin\theta)(z-\cos\theta-j\sin\theta)}{z^5}$

$$H(z) = \frac{(z^3 - z^2 - z + 1)(z^2 - 2z \cos \theta + 1)}{z^5}$$

$$\therefore H(z) = \frac{z^5 - 2z^4 \cos \theta - z^4 + 2z^3 \cos \theta + 2z^2 \cos \theta - z - 2z \cos \theta + 1}{z^5} \dots\dots\dots (11)$$

$$z^5 Y(z) = z^5 X(z) - 2z^4 \cos \theta X(z) - z^4 X(z) + 2z^3 \cos \theta X(z) + 2z^2 \cos \theta X(z) - z X(z) + 2z \cos \theta X(z) + X(z)$$

$$y(n+5) = x(n+5) - 2 \cos \theta x(n+4) - x(n+4) + 2 \cos \theta x(n+3) + 2 \cos \theta x(n+2) - x(n+1) + 2 \cos \theta x(n+1) + x(n)$$

Which is equivalent to:

$$y(n) = x(n) - 2 \cos \theta x(n-1) - x(n-1) + 2 \cos \theta x(n-2) + 2 \cos \theta x(n-3) - x(n-4) + 2 \cos \theta x(n-4) + x(n-5)$$

Or

$$y(n) = x(n) - (1 + 2 \cos \theta)(x(n-1) + x(n-4)) - 2 \cos \theta (x(n-2) + x(n-3)) + x(n-5)$$

The recurrence formula of this filter is

$$\therefore y(n) = x(n) - (1 + 2c)(x(n-1) + x(n-4)) - 2c(x(n-2) + x(n-3)) + x(n-5) \dots\dots (12)$$

Where $c = \cos \theta$

When this filter was applied on EEG data, it was found that although noise artifact and low frequency have somewhat been reduced, but they were still present as shown in figure (8).

An attempt was made to design ,digital filter using as few poles and zeros as possible in order to approximate desired frequency response characteristic since more z-plane poles and zeros are used, the more complicated is resulting recurrence formula and the more numerical calculations are involved in calculating any one out put sample value. Thus, the zero frequency and high frequency rejection are provided by the double zero at $(z = \pm 1)$ and the complex conjugate pole-pair were placed at $z = r e^{\pm j\theta}$ on a circle of radius (r) which gives increased rejection of noise components a round (20) Hz and narrow pass band and a pole-pair on the origin for sharp cut off, as shown in figure (9).

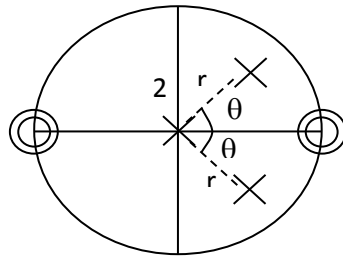


Figure (9): Pole-Zero Configuration of Band Pass Digital Filter with Double Zero at $(z= \pm 1)$, Complex Conjugate Pole-Pair were Placed at $z = re^{\pm j\theta}$ on a Circle of Radius (r) and 2^{nd} Order Poles at Origin.

The z-transform function of this filter was found to be

$$H(z) = \frac{(z-1)(z+1)(z-1)(z+1)}{z^2(z-re^{j\theta})(z-re^{-j\theta})} \dots\dots\dots (13)$$

$$H(z) = \frac{(z^2-1)^2}{z^2(z-r\cos\theta-jr\sin\theta)(z-r\cos\theta+jr\sin\theta)}$$

$$H(z) = \frac{z^4-2z^2+1}{z^4-2z^3r\cos\theta+z^2r^2} = \frac{Y(z)}{X(z)} \dots\dots\dots (14)$$

$$z^4Y(z)-2z^3r\cos\theta Y(z)+z^2r^2Y(z)=z^4X(z)-2z^2X(z)+X(z)$$

Then the recurrence formula becomes

$$y(n+4)-2r\cos\theta y(n+3)+r^2y(n+2)=x(n+4)-2x(n+2)+x(n)$$

Which is equivalent to:

$$y(n)-2r\cos\theta y(n-1)+r^2y(n-2)=x(n)-2x(n-2)+x(n-4)$$

The recurrence formula of this filter is.

$$y(n)=2r\cos\theta y(n-1)-r^2y(n-2)+x(n)-2x(n-2)+x(n-4) \dots\dots\dots (15)$$

The pole-position used were at $(\theta = \frac{\pi}{6})$ and $(r = 0.4)$.

Figure (10) shows the result when this filter was applied to EEG data. The noise reduction in the signal was very good, when $(\theta = \pi/6)$ and $(r = 0.2)$ which represent the best local (angle and distance) for a pairs of poles, but some slow wave can be seen.

Figure (11) shows the output of filter of figure (9) with $(\theta = \frac{\pi}{4})$ and $(r = 0.4)$, it is clear that there is some noise and artifact couldn't be removed.

In this filter, double zero are placed at $(z = \pm 1)$ and single pole placed on a circle of radius (r_1 and r_2) and a pole-pair on the origin for sharp cut off was designed ,as shown in figure (12).

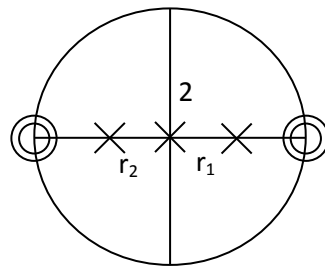


Figure (12): Pole-zero Configuration of Band Pass Digital Filter with Double Zero are Placed at ($z = \pm 1$), Single Pole Placed on a Circle of Radius (r_1 and r_2) and a Pole-Pair on the Origin.

The z-transform function of this filter was found to be

$$H(z) = \frac{(z-1)(z+1)(z-1)(z+1)}{z^2(z-r_1)(z+r_2)} \text{----- (16)}$$

$$H(z) = \frac{(z^2-1)^2}{z^2(z^2-r_1z+r_2z-r_1r_2)}$$

$$H(z) = \frac{z^4-2z^2+1}{z^4-r_1z^3+r_2z^3-r_1r_2z^2} \text{----- (17)}$$

$$z^4Y(z) - r_1z^3Y(z) + r_2z^3Y(z) - r_1r_2z^2Y(z) = z^4X(z) - 2z^2X(z) + X(z)$$

Then the recurrence formula becomes

$$y(n+4) - r_1y(n+3) + r_2y(n+3) - r_1r_2y(n+2) = x(n+4) - 2x(n+2) + x(n)$$

Which is equivalent to:

$$y(n) - r_1y(n-1) + r_2y(n-1) - r_1r_2y(n-2) = x(n) - 2x(n-2) + x(n-4)$$

The recurrence formula of this filter is.

$$y(n) = r_1y(n-1) - r_2y(n-1) + r_1r_2y(n-2) + x(n) - 2x(n-2) + x(n-4) \text{----- (18)}$$

When this filter was applied with ($r_1=0.8$ & $r_2=0.3$) to EEG data. As shown in figure (13), it was found that the noise has not been completely eliminated.

By varying the value of (r_1 & r_2) of the filter of figure (12) and applied on the same EEG data, the noise and artifacts components were also not completely rejected as shown in figure (14) with ($r_1=0.2$ & $r_2=0.9$).

Finally ,the pervious filter can be modified to reduce the noise components completely by applying double zero at ($z = \pm 1$), single pole placed on a circle of radius (r_1 and r_2) and 4th order pole was placed at the origin to provide sharp cut off as shown in figure (15) .

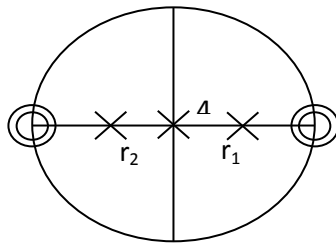


Figure (15): Pole-Zero Configuration of Band Pass Digital Filter with double zero at $(z = \pm 1)$, single pole placed on a circle of radius $(r_1$ and $r_2)$ and 4^{th} order pole was placed at the origin.

The z-transform function of this filter was found to be

$$H(z) = \frac{(z-1)(z+1)(z-1)(z+1)}{z^4(z-r_1)(z+r_2)} \text{----- (19)}$$

$$H(z) = \frac{z^4 - 2z^2 + 1}{z^6 - r_1z^5 + r_2z^5 - r_1r_2z^4} = \frac{Y(z)}{X(z)} \text{----- (20)}$$

$$z^6Y(z) - r_1z^5Y(z) + r_2z^5Y(z) - r_1r_2z^4Y(z) = z^4X(z) - 2z^2X(z) + X(z)$$

Then the recurrence formula becomes

$$y(n+6) - r_1y(n+5) + r_2y(n+5) - r_1r_2y(n+4) = x(n+4) - 2x(n+2) + x(n)$$

Which is equivalent to:

$$y(n) - r_1y(n-1) + r_2y(n-1) - r_1r_2y(n-2) = x(n-2) - 2x(n-4) + x(n-6)$$

Or

$$y(n) = r_1y(n-1) - r_2y(n-1) + r_1r_2y(n-2) + x(n-2) - 2x(n-4) + x(n-6) \text{----- (21)}$$

When this filter was applied with $(r_1=0.4$ & $r_2=0.7)$, a reduction in the noise components and low frequencies was obtained as shown in figure (16). To be sure that the noise components was reduced accurately, another two applications of the above filter were applied to the same EEG data one with $(r_1=0.9$ & $r_2=0.1)$ and the other with $(r_1=0.1$ & $r_2=0.8)$.The results of these applications are shown in figure (17) and figure (18) respectively. In comparison between there figures, it was found that the filter with $(r_1=0.1$ & $r_2=0.8)$ yielded a useful reduction in noise components together with spikes

which are sufficiently clear to be detected by means of a simple threshold technique.

C-Threshold Technique for Detection of the Spike Sequence

In order to study the incidence patterns of the contraction spike they must be recognized and separated from the noise and presented as an event sequence in time. That is to say , a new signal must be generated which has the value **0** at all times except the sample point at which a spike is recognized ,when its value must be **1** .Such signal is known as a point event series and it is

obtained from the filtered spike signal by means of a threshold detection algorithm.

Several factors contribute to the design of such a routine. In our spike signal the amplitude of the spike is generally greater than all other components so in principle a simple threshold system will detect its presence.

Figure (19) shows the extraction process applied to one minute of EEG (channel two). The top trace (a) shows the original EEG record for one minute. The center trace (b) shows the signal after being filtered with the a band pass digital filter whose z-plane plot is shown on figure (15). The trace (c) shows the binary out put from the threshold program; it is the point process representation of the spike component incidence. It is clear from this figure that the incidence of spike has been successfully recognized.

D-Bar Chart

The bar chart is a convenient graphical device that is particularly useful for displaying nominal or ordinal data-data like ethnicity, sexes and treatment category. The various categories are represented along the horizontal axis. The height of each bar is equal to the frequency of items for the category [7]. Bar charts are usually drawn with a gap between the bars (rectangles), because each bar describes a different item [3].

Bar Chart Applied on EEG Data

The bar chart routine was applied to EEG data of normal subject, as shown in figure (20). The top trace (a) shows the original EEG record for one minute of EEG data. Trace (b) shows the results of filtered signal by using band pass digital filter of figure (15). The third trace (c) shows the point event

representation of the spikes. The bottom trace (d) shows the number of spikes per second for one minute generated by program (**Bar chart**). Each bar represents one second. The height of the bar equals to the number of spikes per second.

As a check on the bar chart operation "by eye" count the number of spikes showed that the bar chart truly represented the number of spikes per second incidence patterns in the original data.

Discussion

Computer analysis of EEG aims to extract information from the signal and presents it in a more objective and convenient for interpretation. Many analytical techniques, such as spectral analysis standard deviation, have been developed. These techniques were based on their analysis on Fast Fourier Transform (FFT). Disadvantages with spectral analysis are the requirement of a fairly long observation time to achieve good spectral estimates. Another disadvantage is that the power spectrum doesn't give the desired result, when a certain characteristic values are need, like peak frequencies.

In the present study, a new technique was developed and applied on one minute EEG record of normal and epileptically females and male. The develop band pass-threshold-bar chart technique was based on z-transform signal process. This technique gives an accurate measure of the spike incidence patterns in EEG data. The problem of noise, slow wave and artifacts component can be effectively eliminated.

In addition, the development technique in actually counting spikes and displaying their incidence patterns is a great important on the technique. In which an adaptive spike

detection algorithm was constructed by combining the different threshold value of discriminate function.

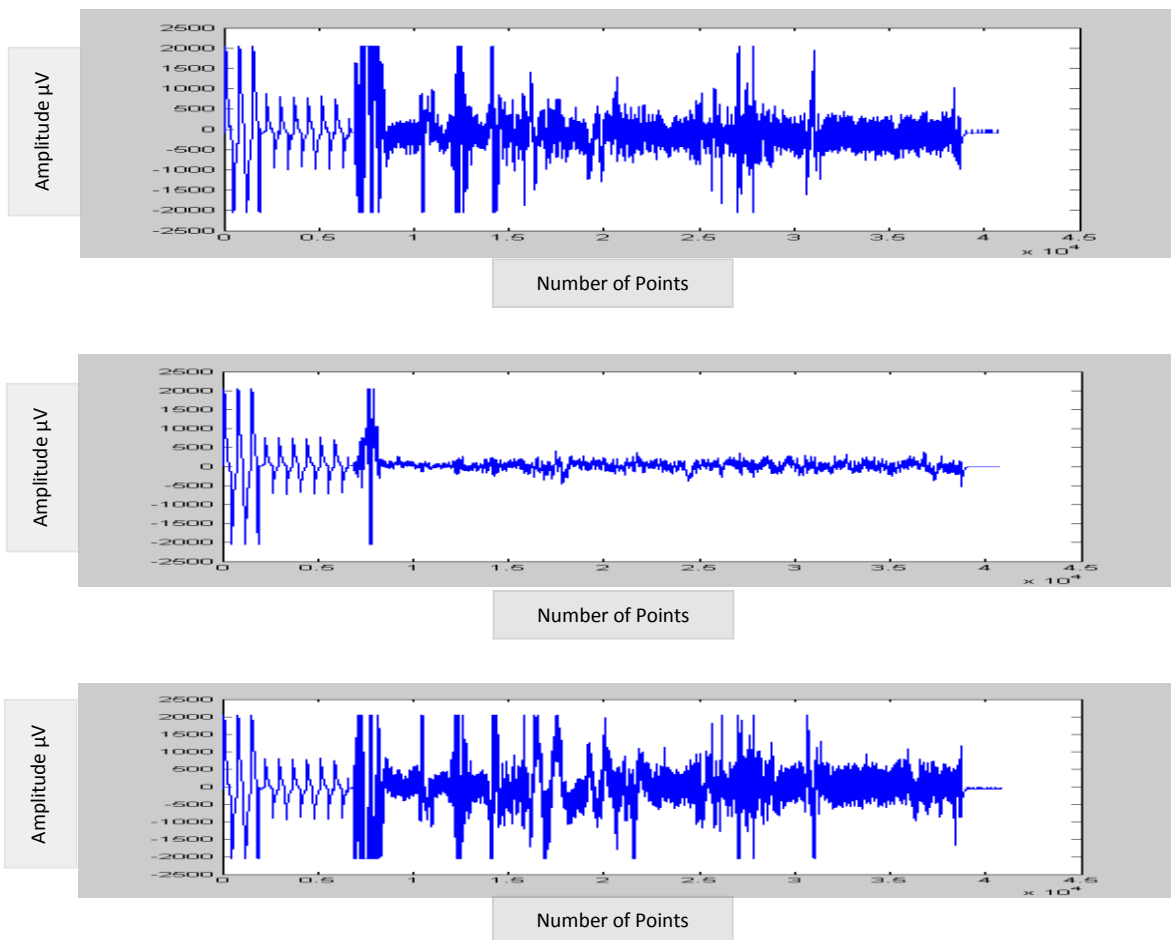
Application the band pass threshold technique on EEG data for one minute duration of normal and epileptically female and male during open and closed eyes state, the application of this technique shows an increase in the number of spikes per second of epileptically female and male as compared to the EEG of normal female and male this increase reface to more action potential (an electrical phenomenon that occurs in nerve cells) occurs.

Conclusion

In EEG assessment, transient activities mixed with background activity play an important

role in neurology. Spikes are sometimes hidden to the eye dominate noise. So, it was necessary to develop a technique of a great complexity to extract the spikes from slow wave and noise components. Thus, with original real time sampling of 160 Hz spikes could be adequately extracted by means of a band pass filter has a double zero at $z=\pm 1$, single pole placed on a circle of radius (r_1 and r_2) and 4th order pole was placed at the origin. After this filtering operation spikes incidence could be recognized using a simple threshold routine. The output of threshold routine was used successfully to generate a bar chart to calculate the number of spikes per second.

It is concluded that this method is potentially very useful in the analysis of EEG signal.



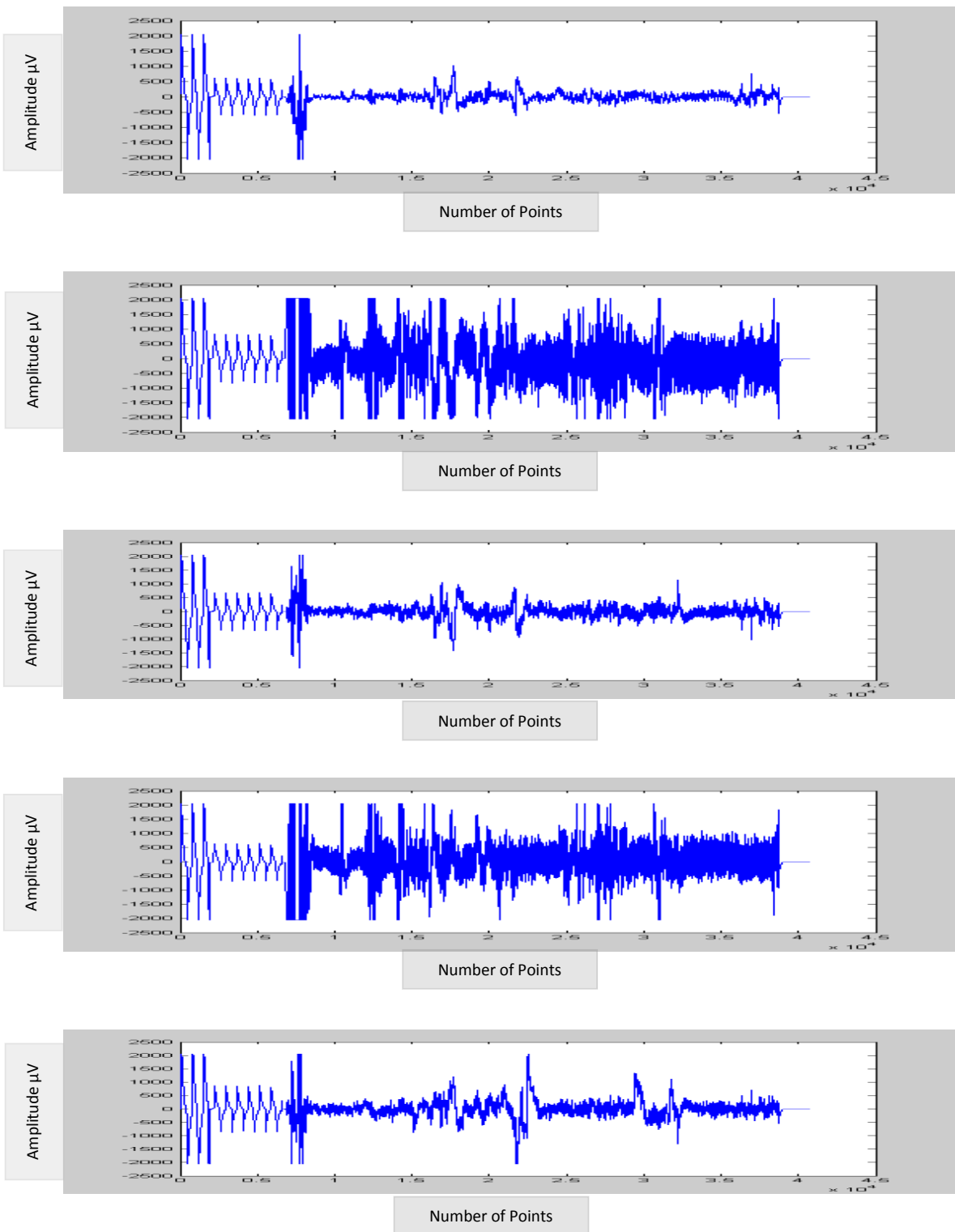
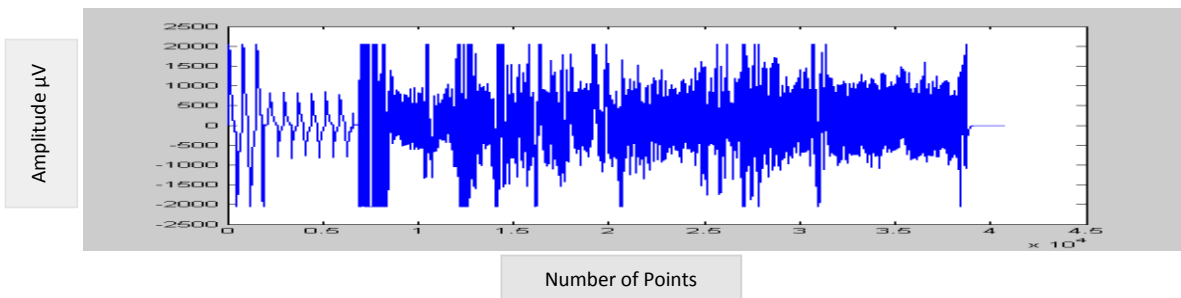
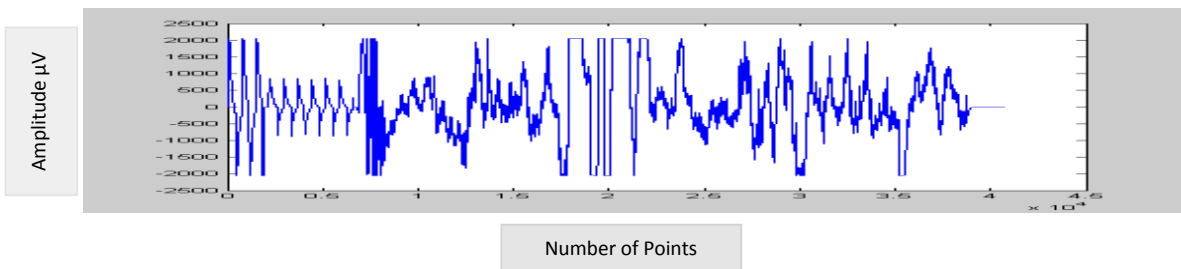
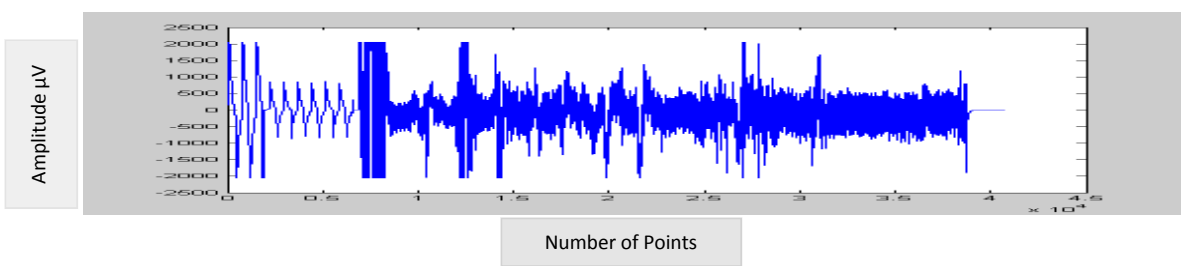
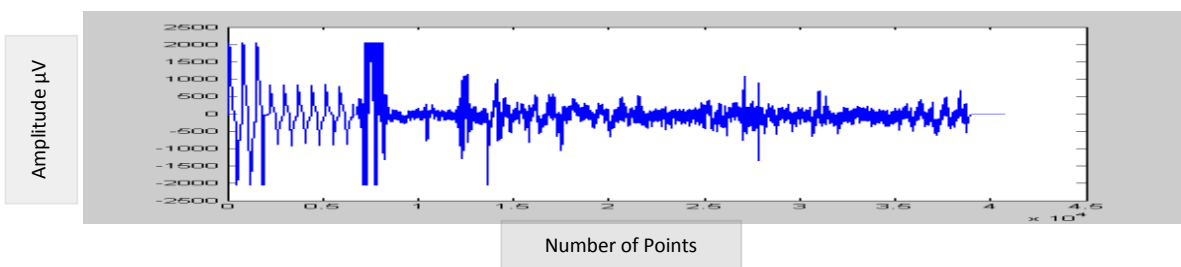
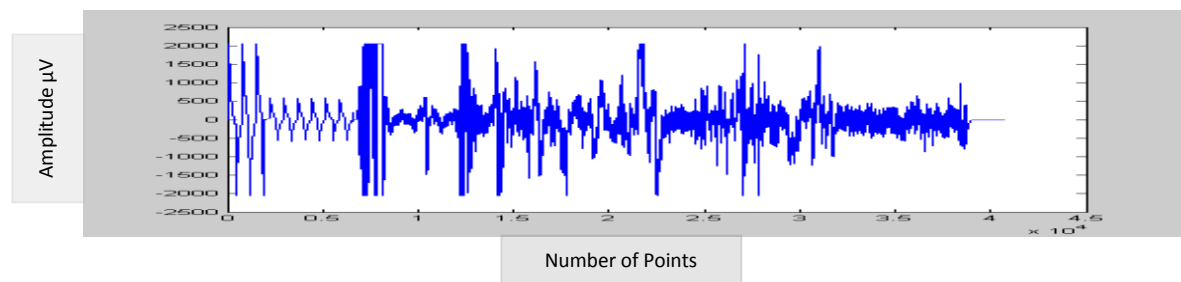
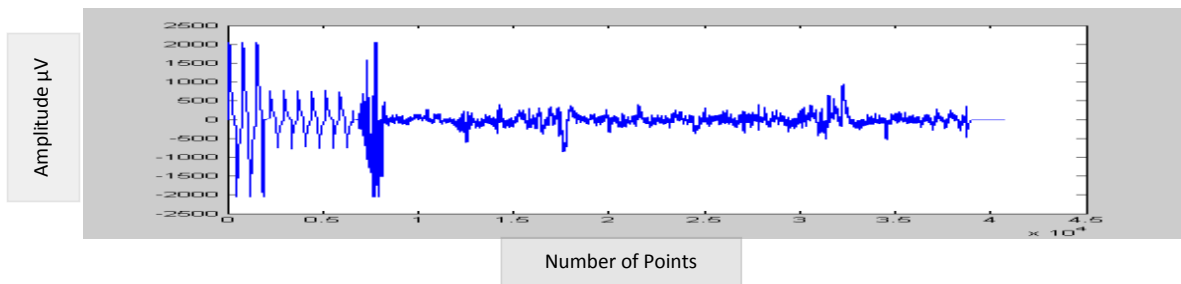


Figure (3a): First 8th Channels of EEG Signal of Four Minutes Duration



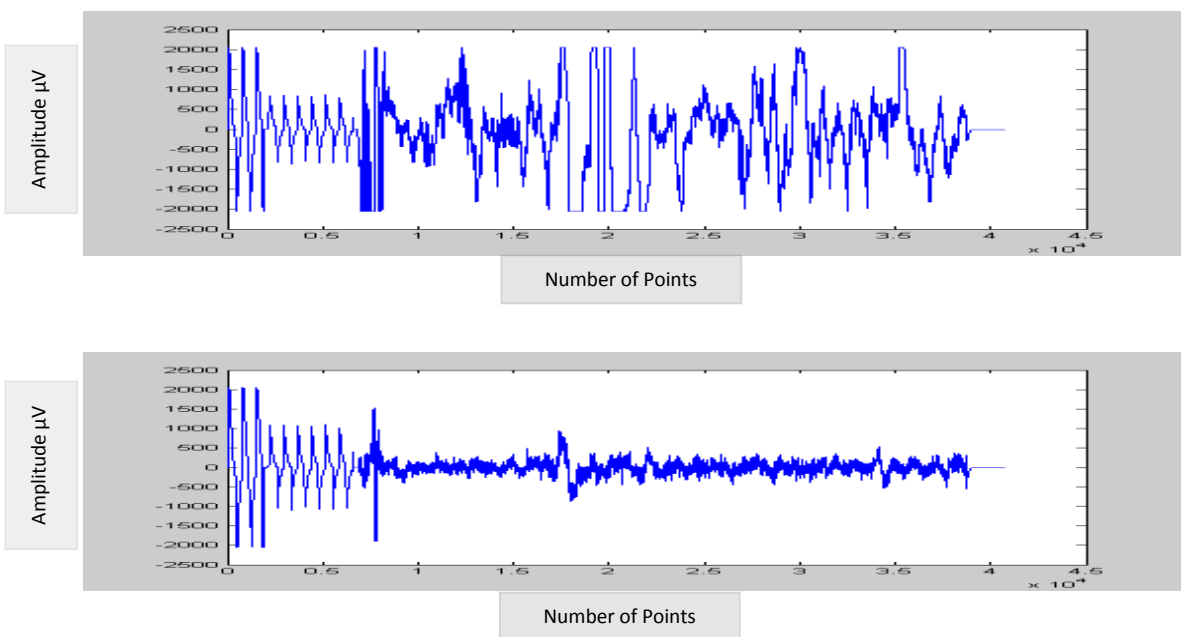
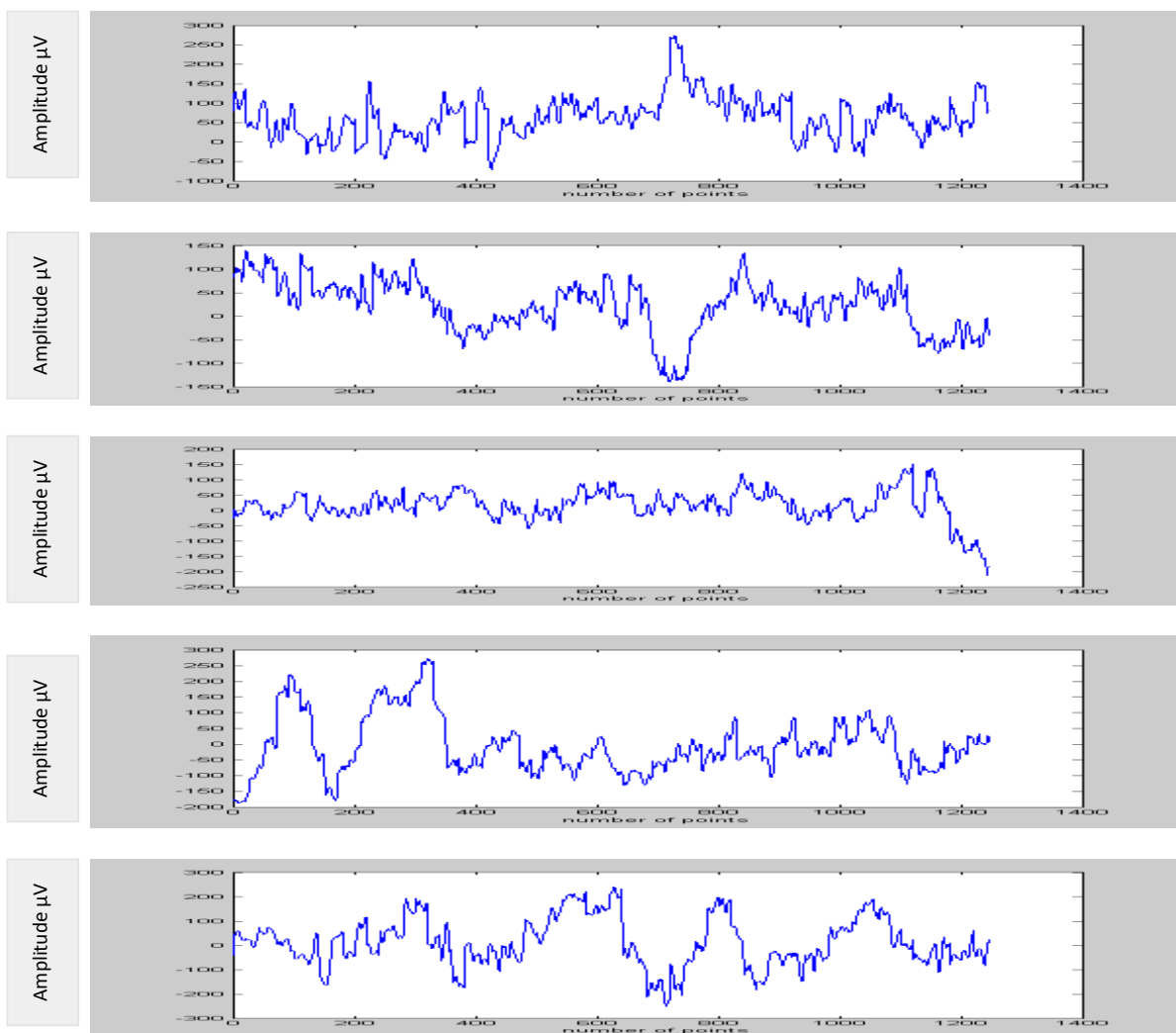


Figure (3b): Second 8th Channels of EEG Signal of Four Minutes Duration



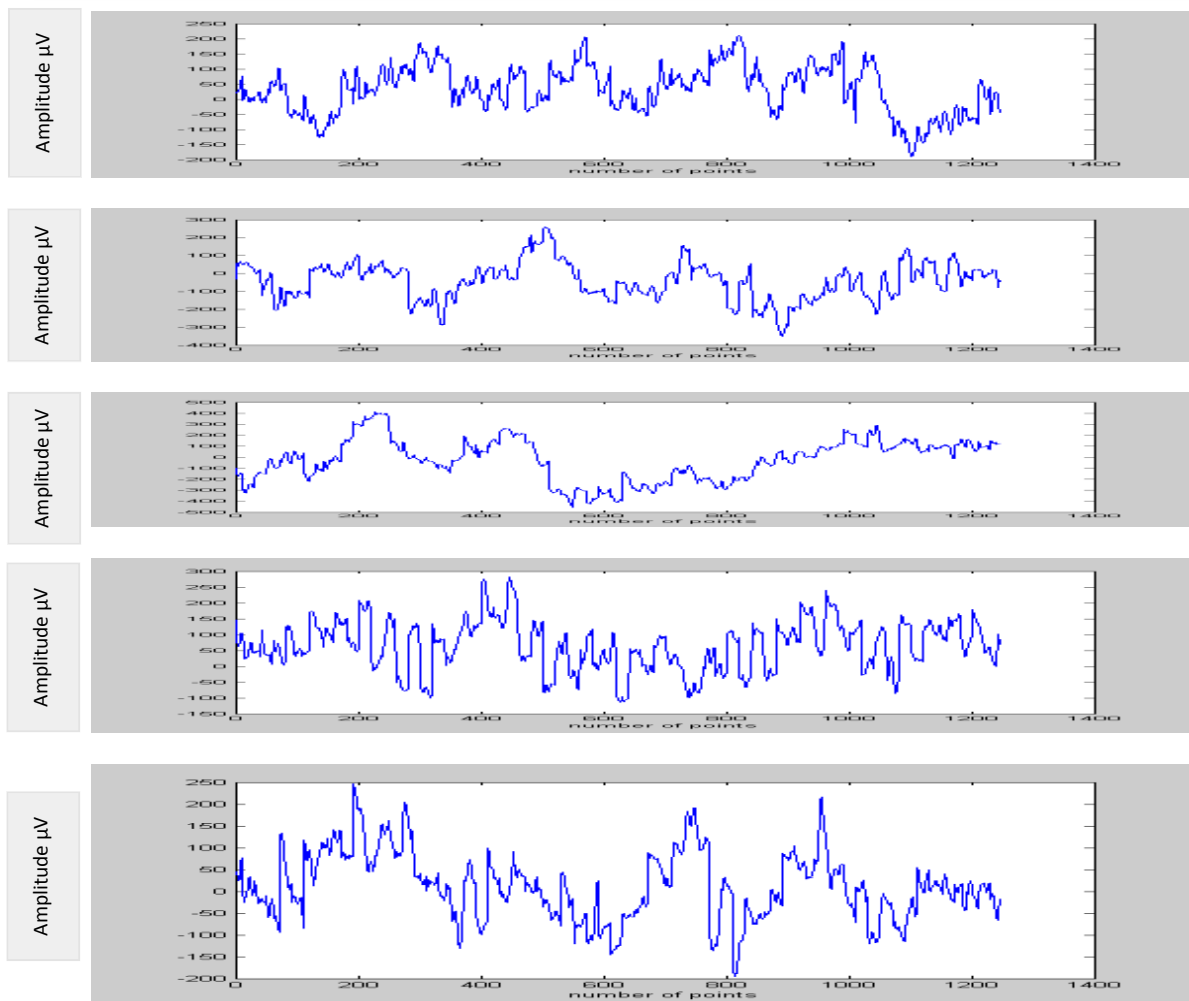
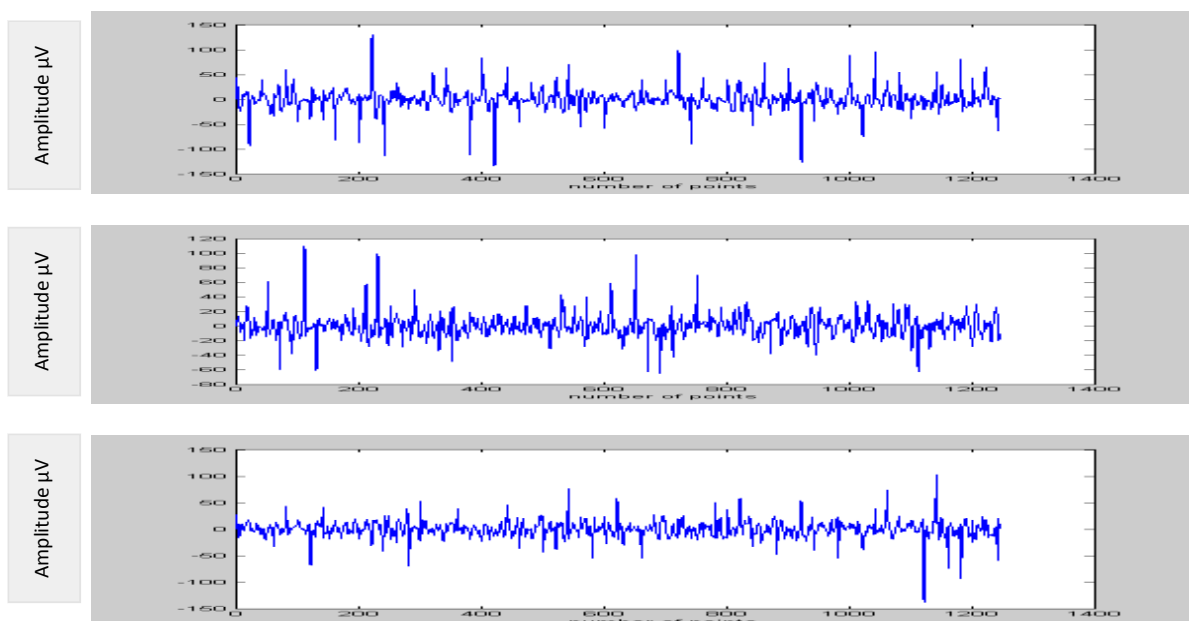


Figure (4):Ten Traces of Channel Two of One Minute off EEG1250 Points on Each Tracer to Six Second Duration Corresponding



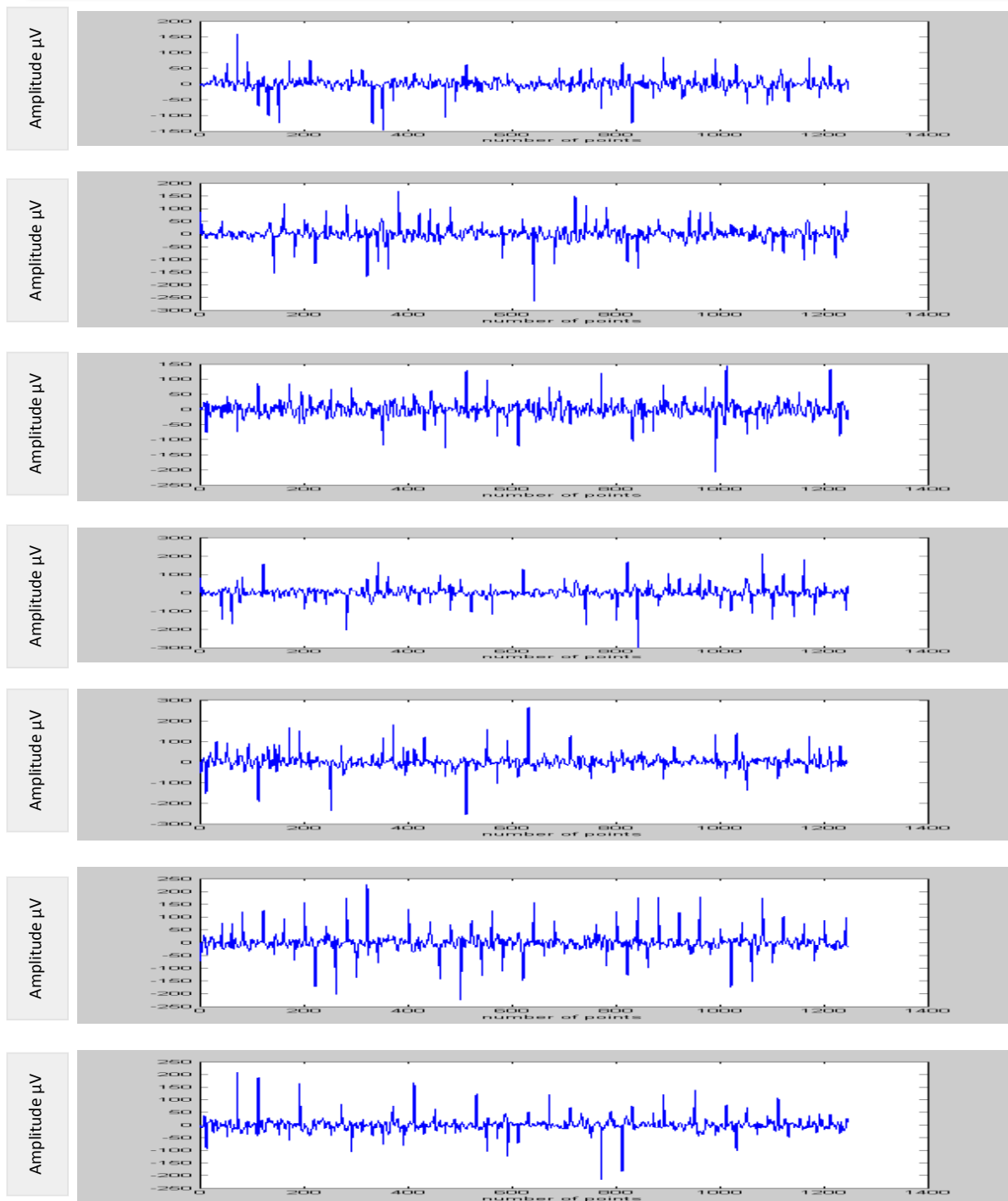
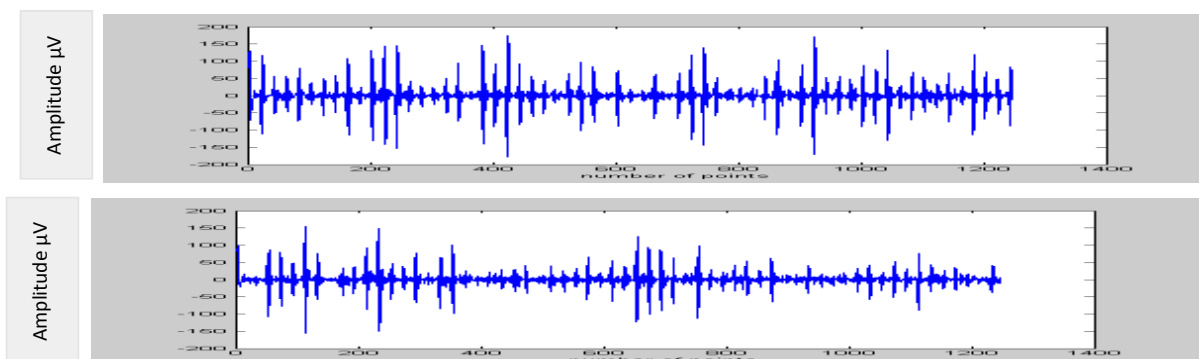


Figure (6): Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (5)



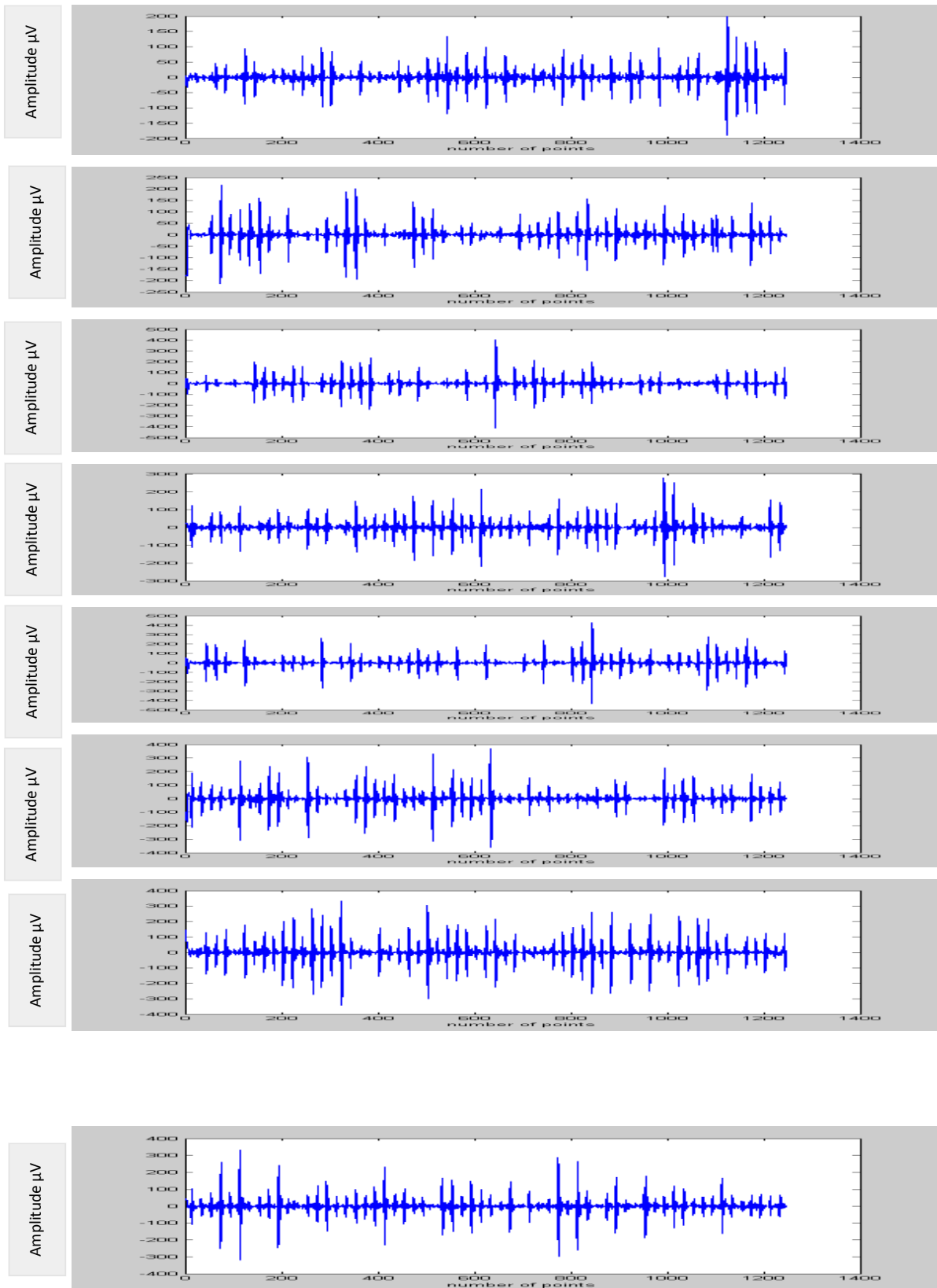
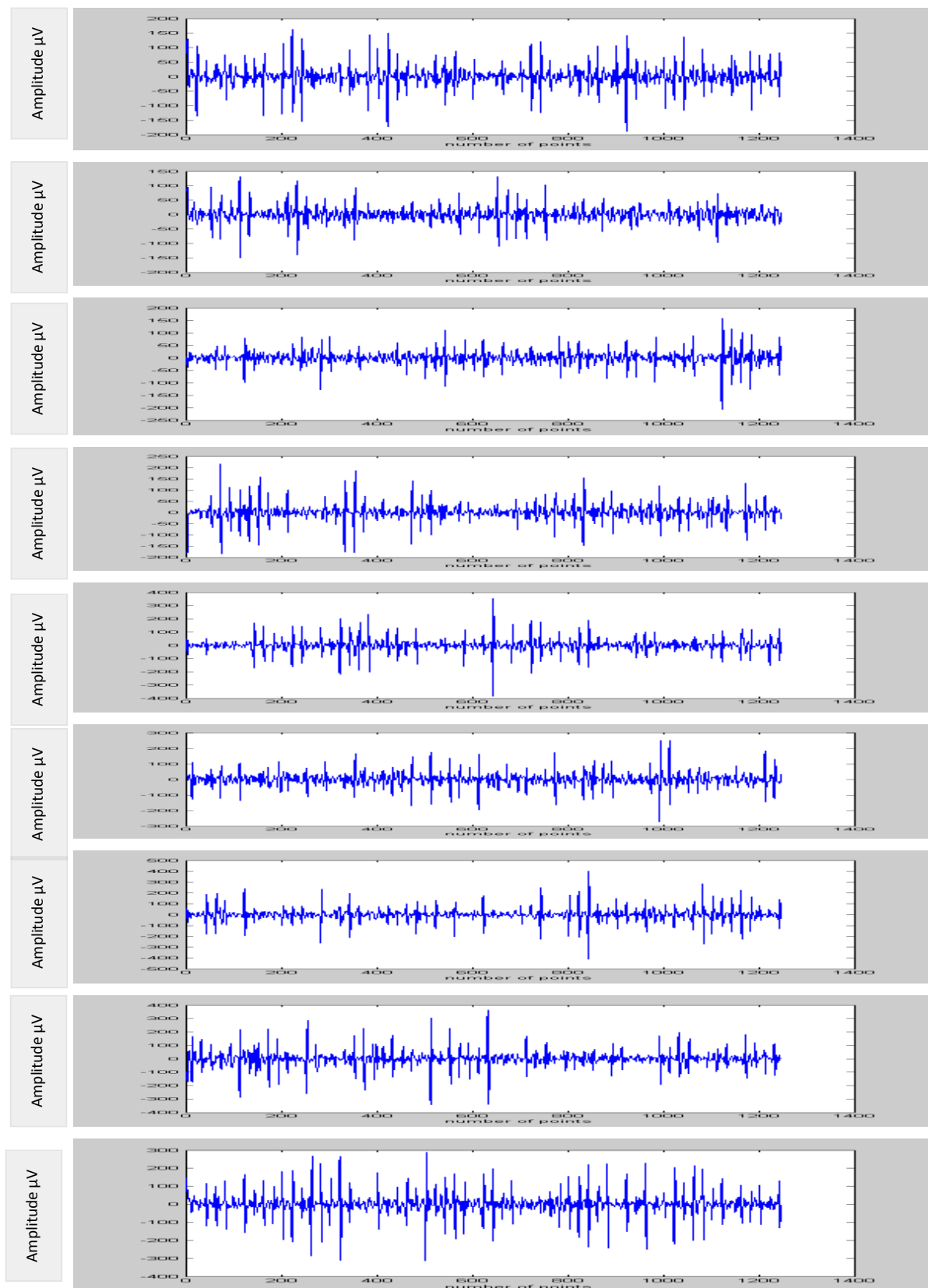


Figure (8): Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (7)



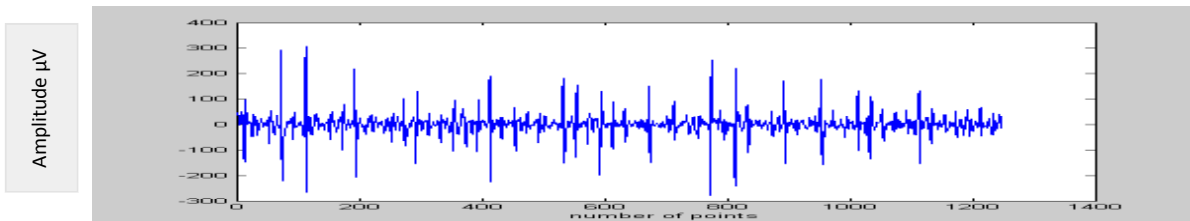
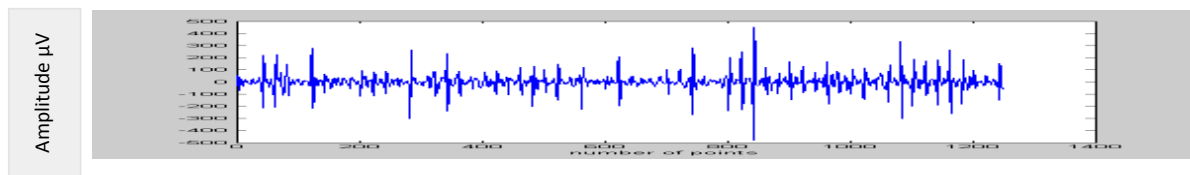
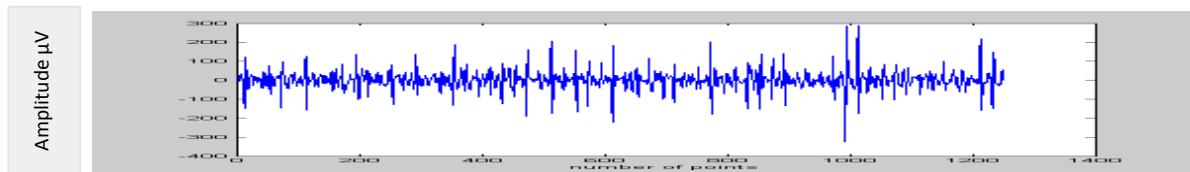
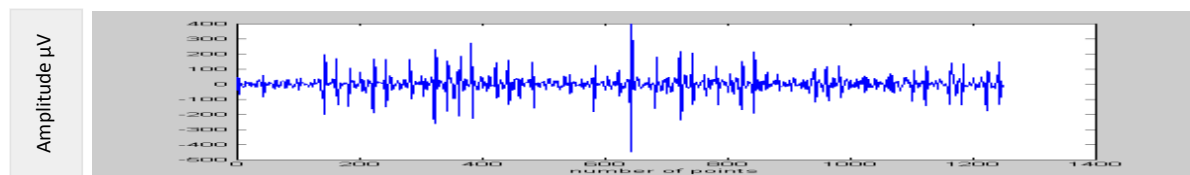
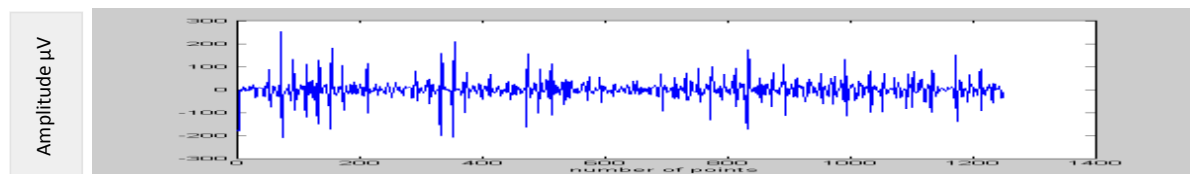
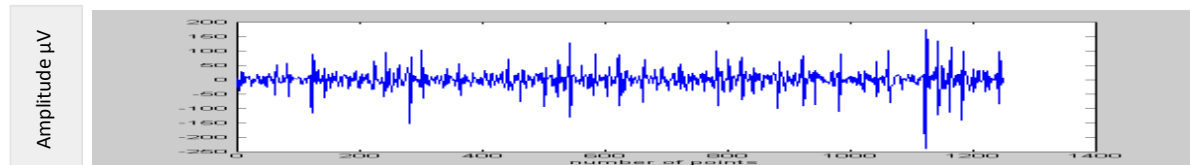
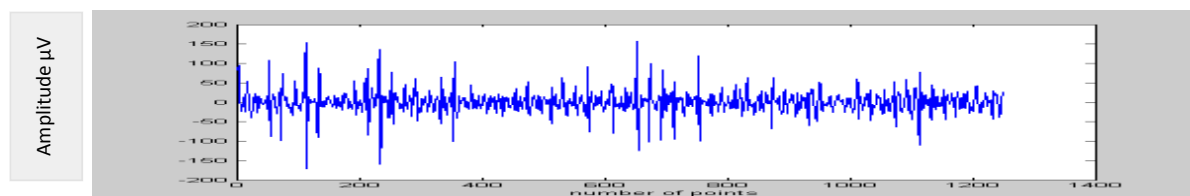
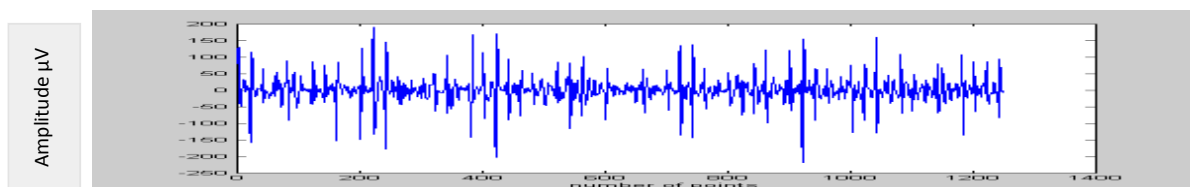


Figure (10) : Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (9)with $(\theta= \pi /6) , (r=0.2)$

filter of figure (9)with $(\theta= \pi /6) , (r=0.2)$



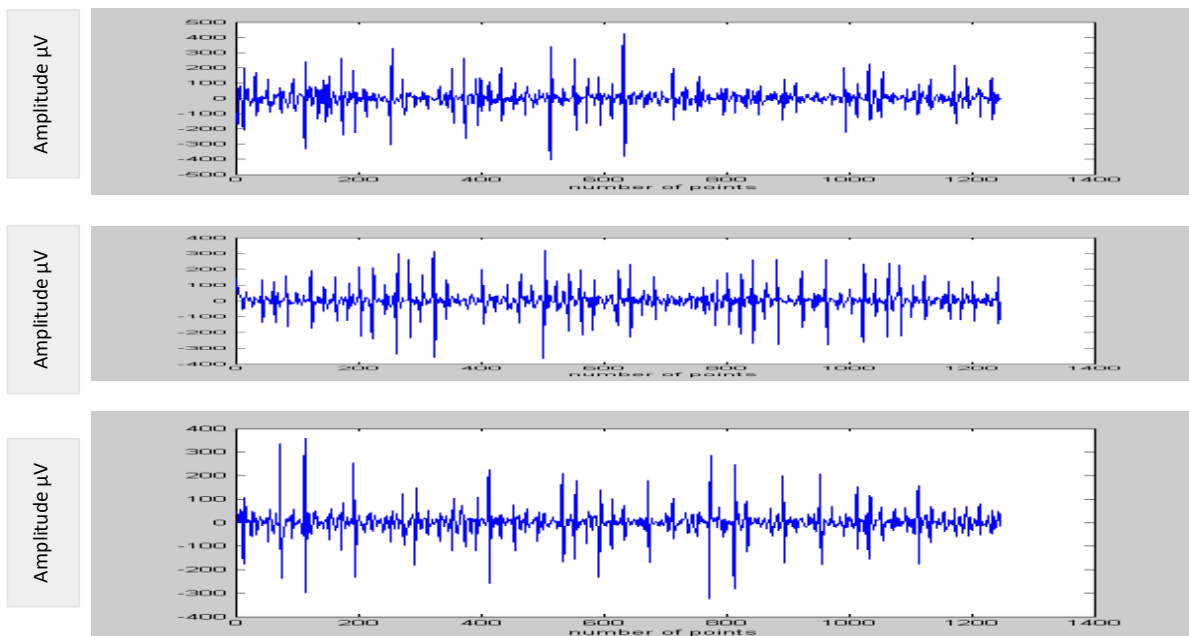
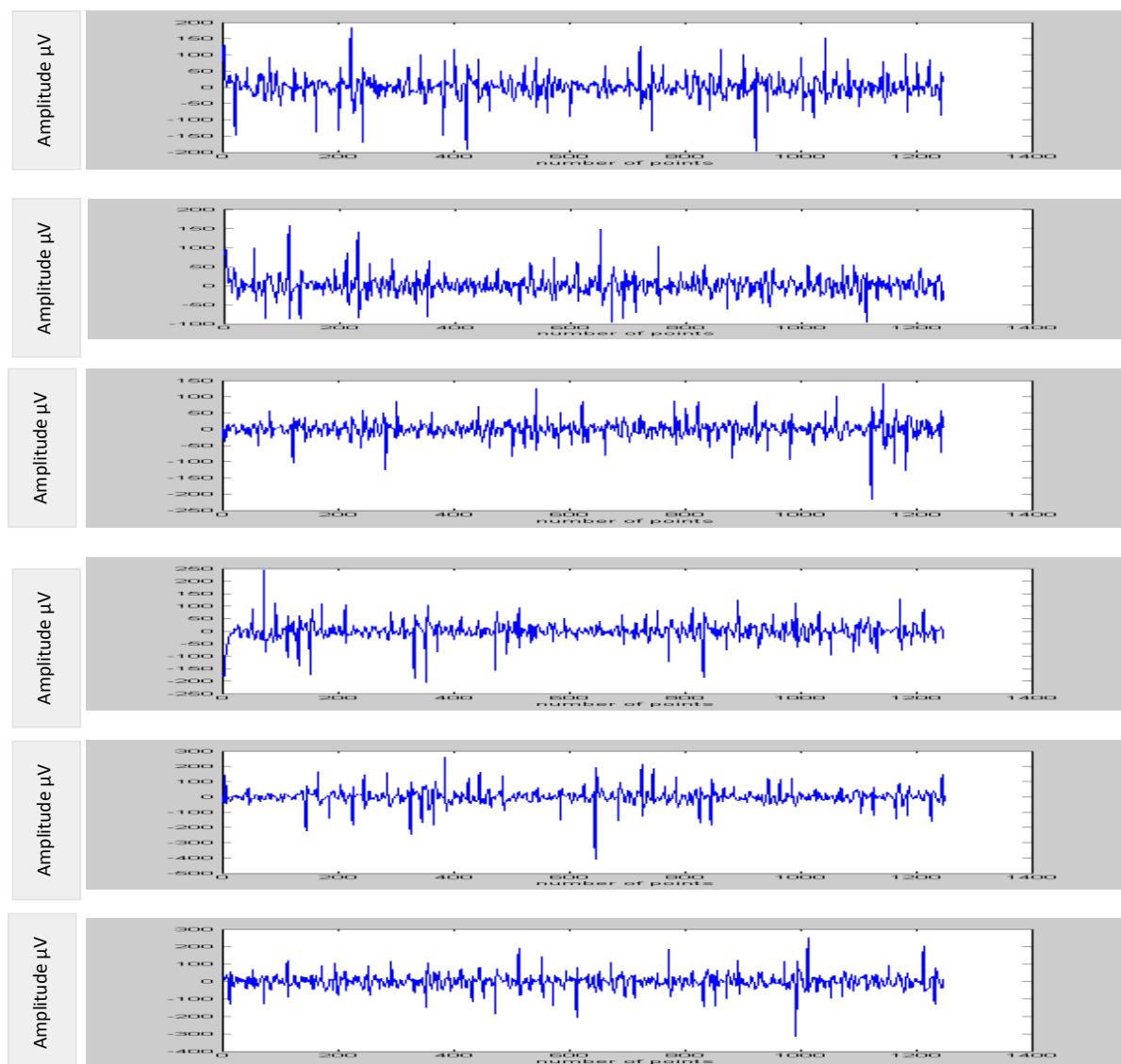


Figure (11) : Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (9) with $(\theta = \pi / 4)$, $(r = 0.4)$



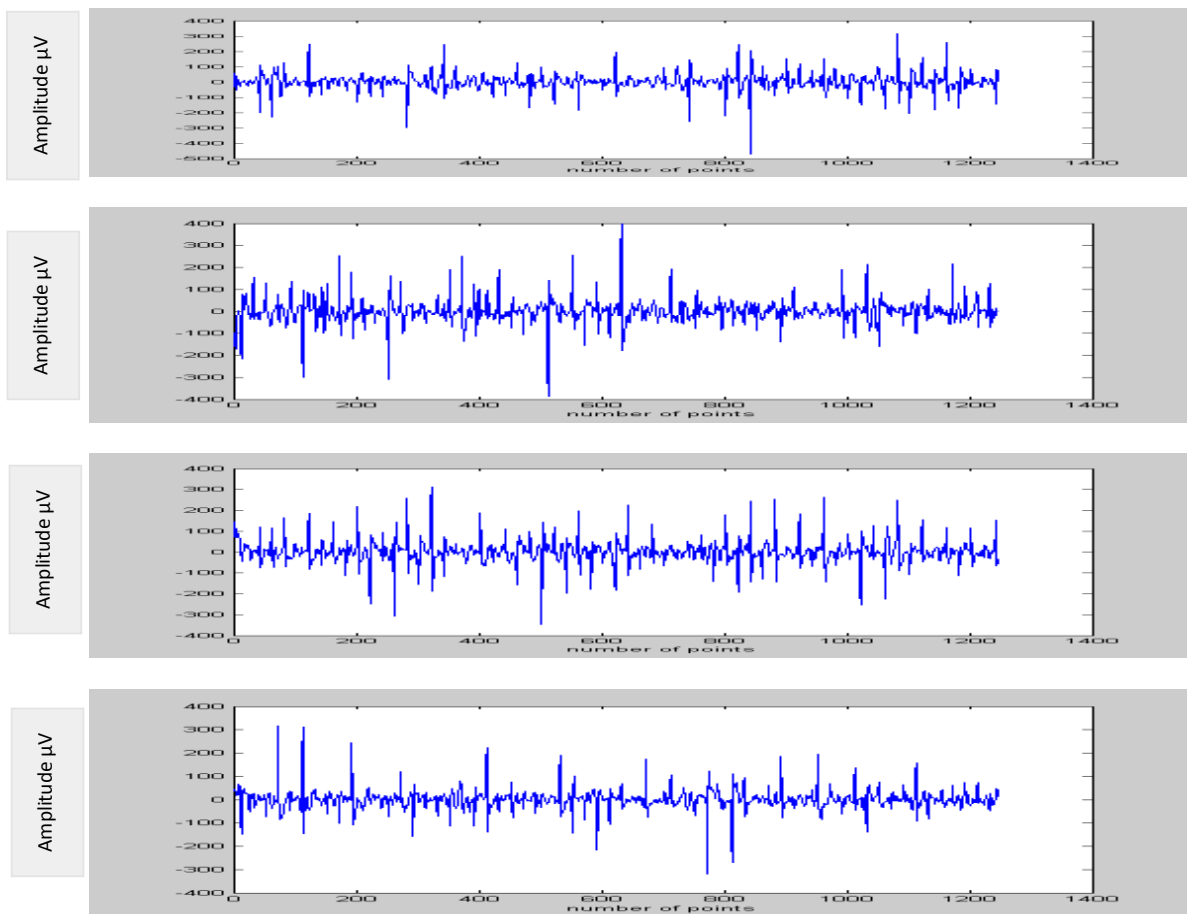
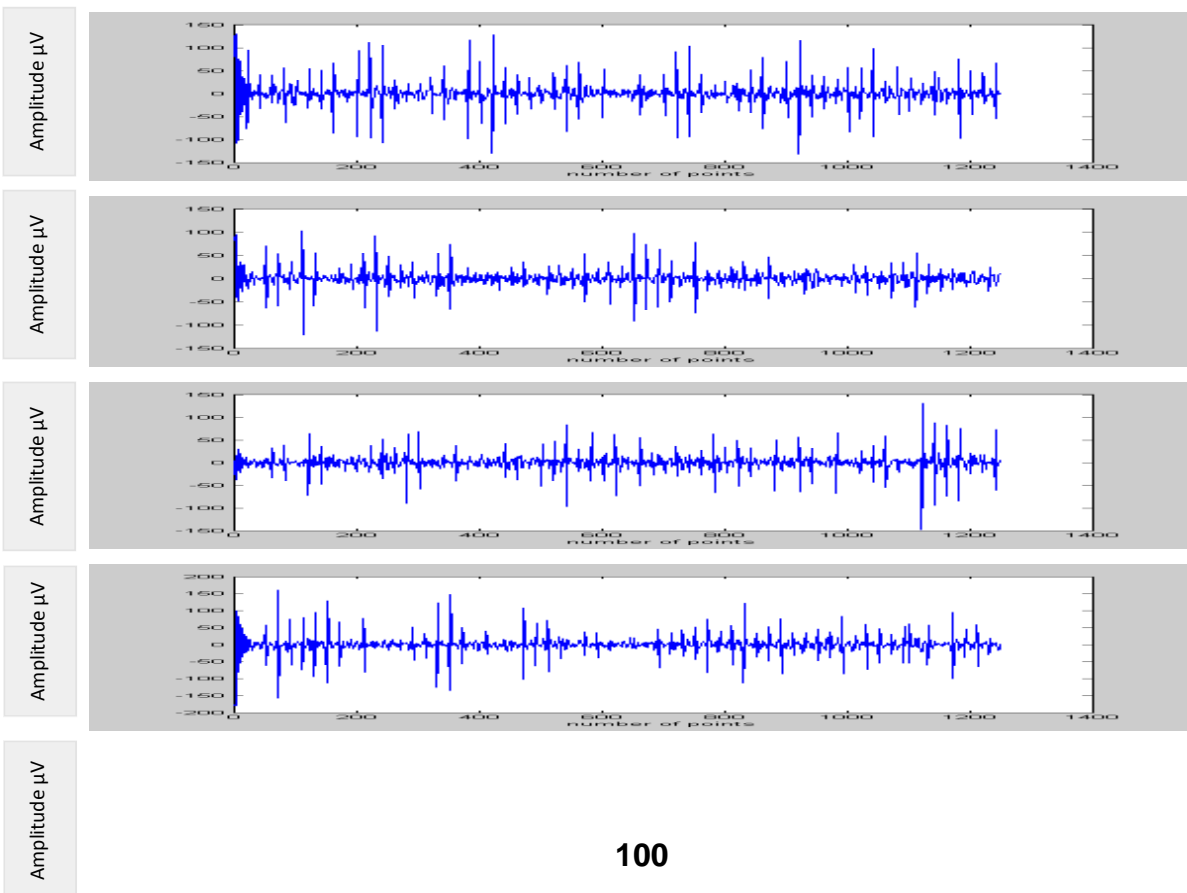


Figure (13) : Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (12) with ($r_1=0.8$ & $r_2=0.3$)



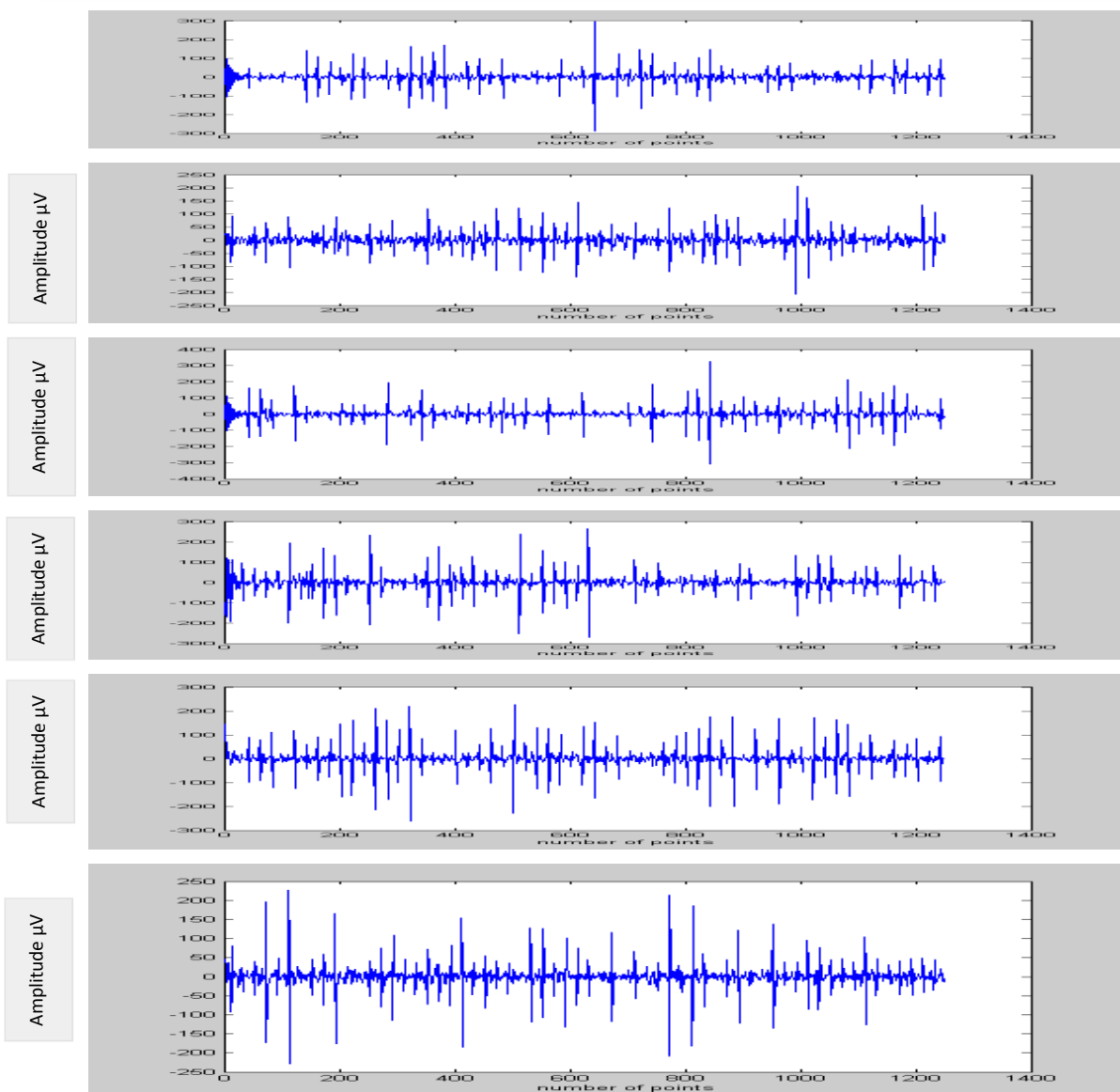
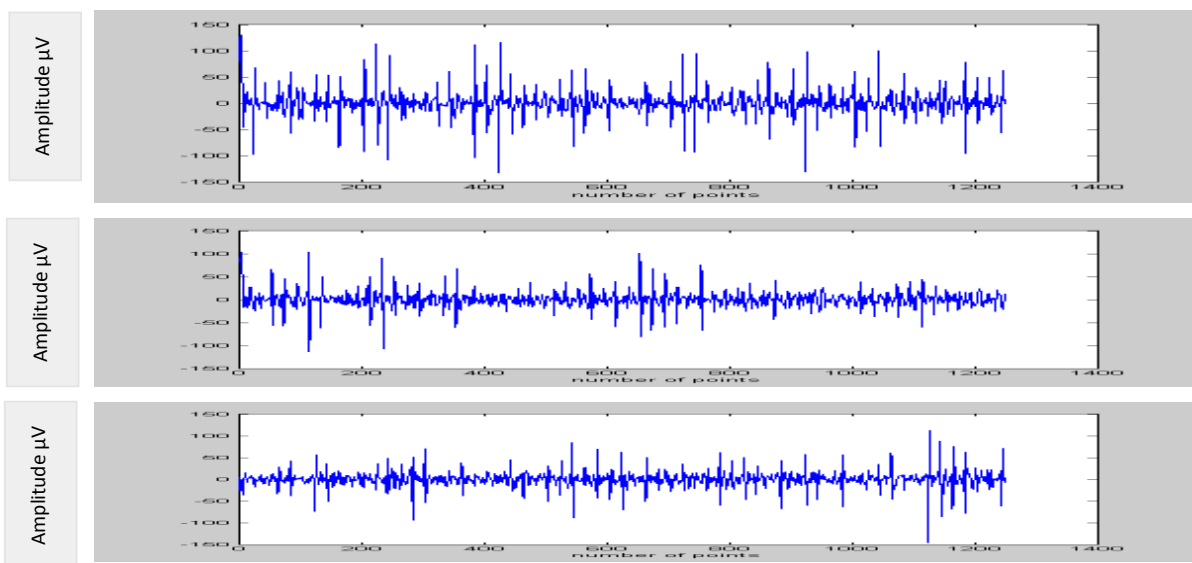


Figure (14): Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (12) with ($r_1=0.2$ & $r_2=0.9$)



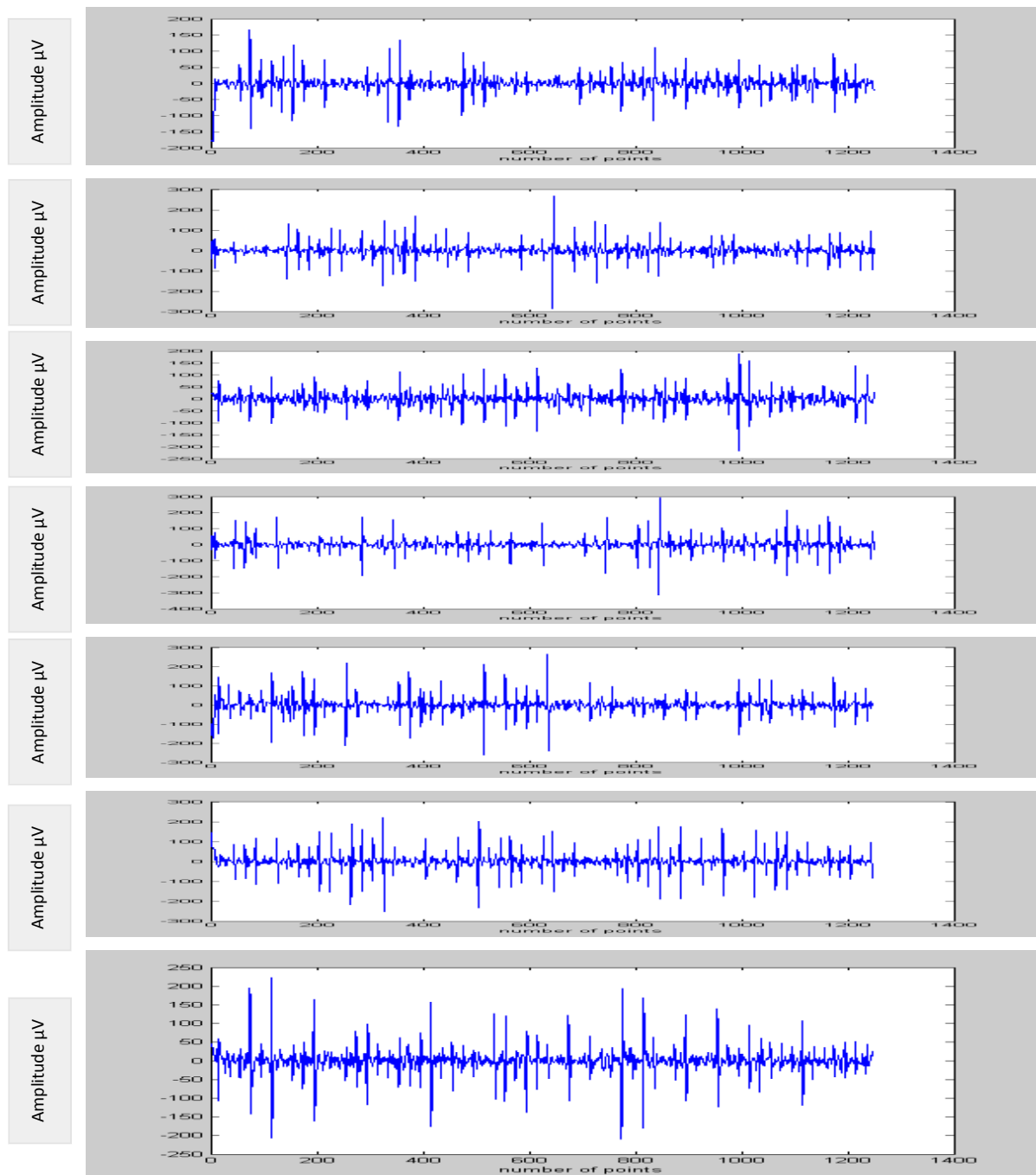
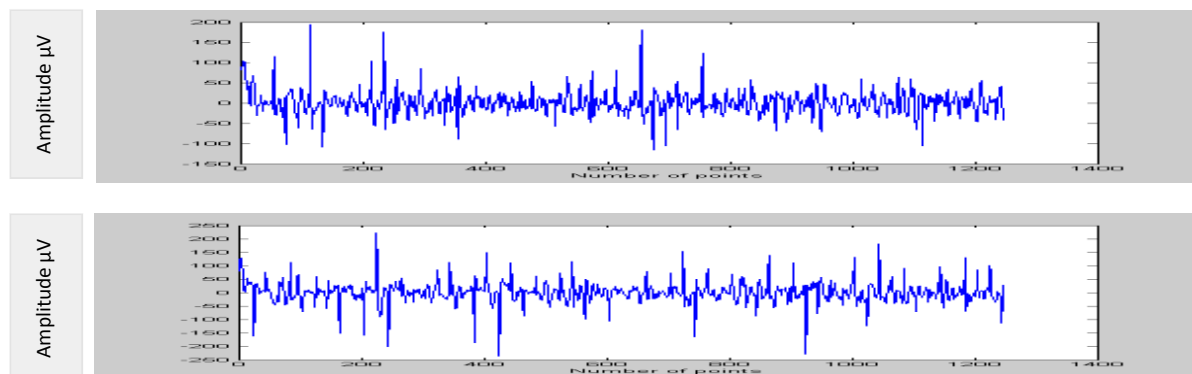


Figure (16) : Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (15) with ($r_1=0.4$ & $r_2=0.7$)



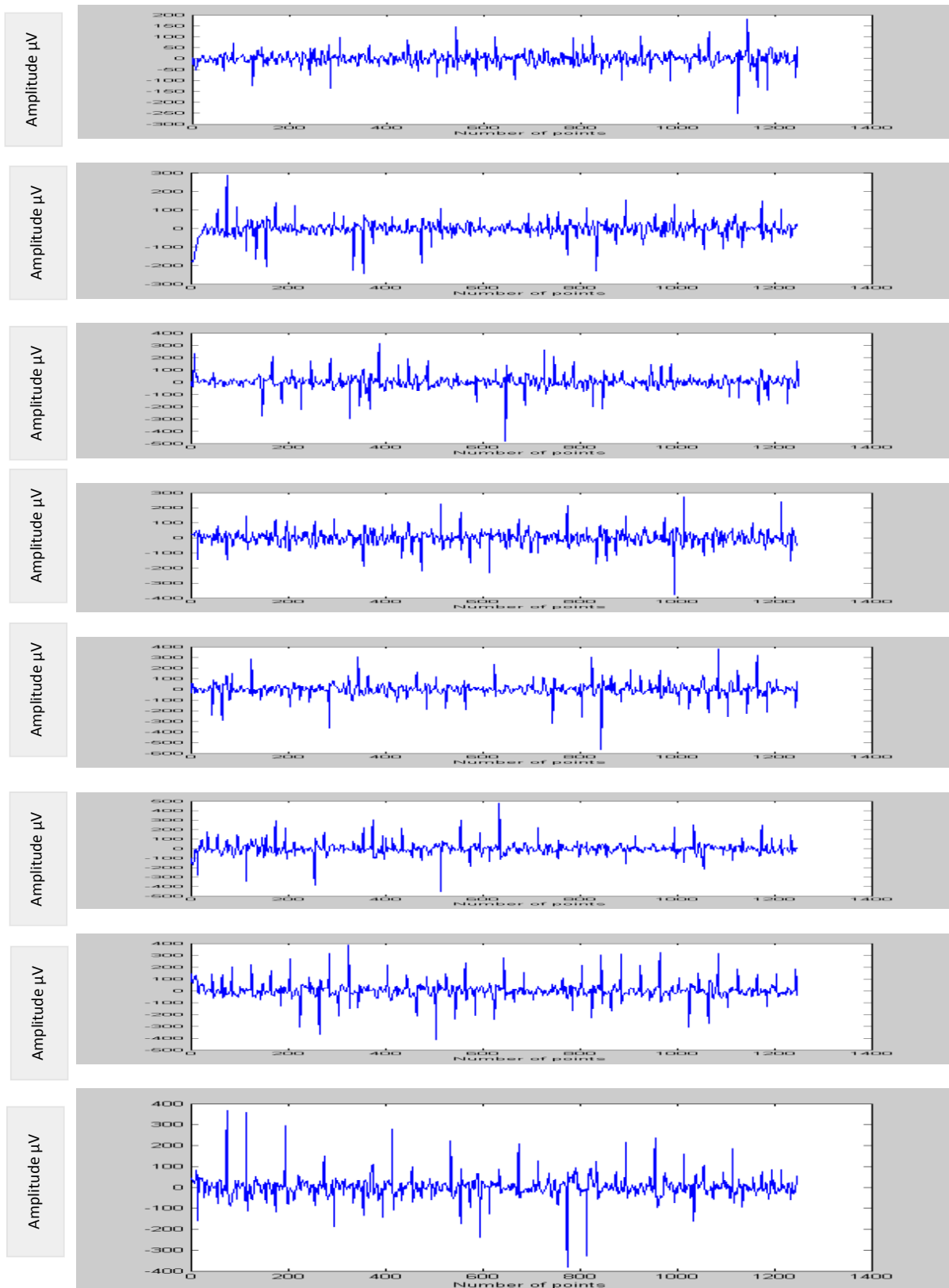
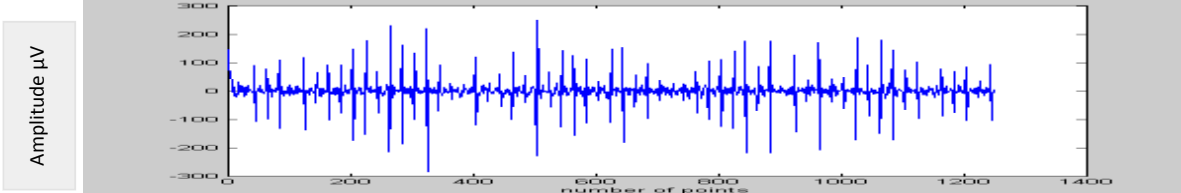
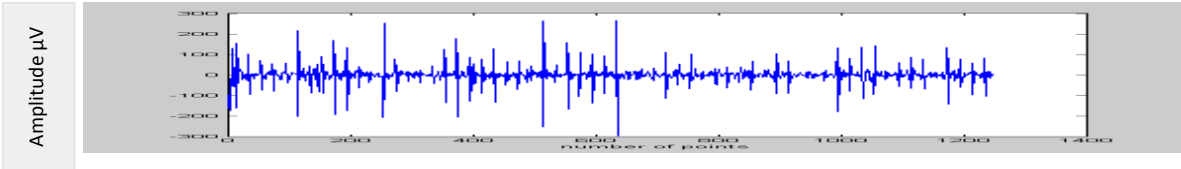
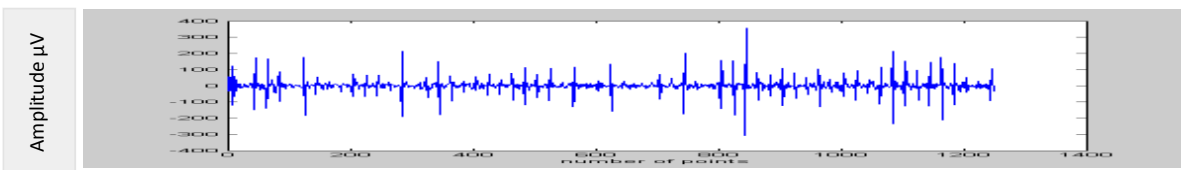
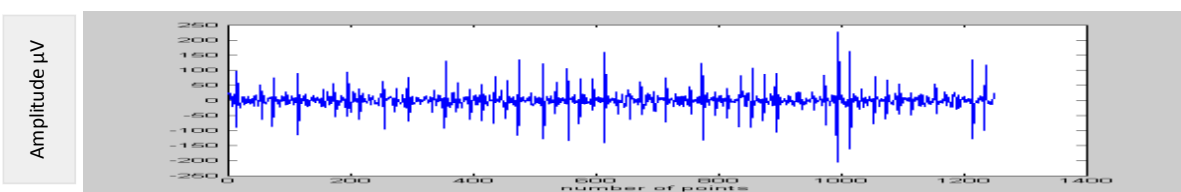
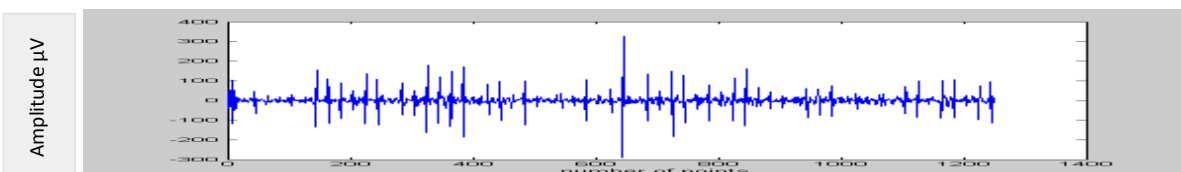
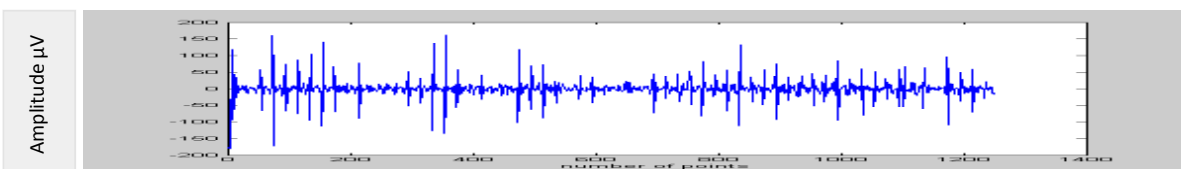
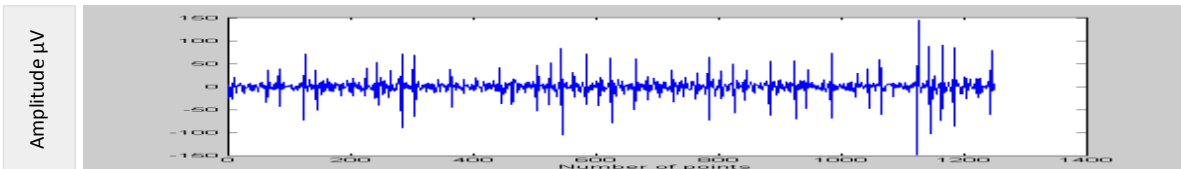
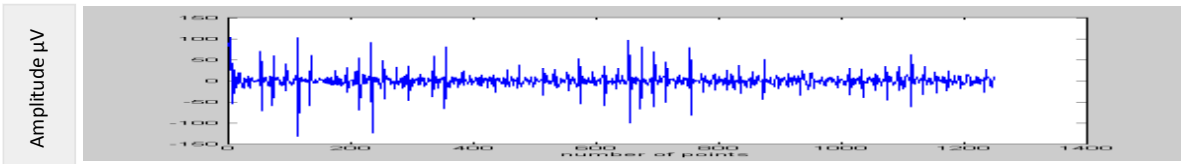
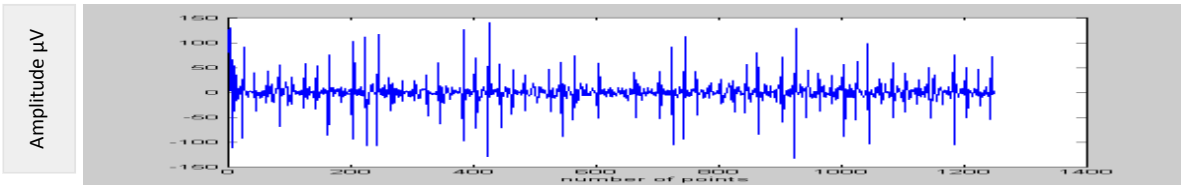


Figure (17) : Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (15) with ($r_1=0.9$ & $r_2=0.1$)



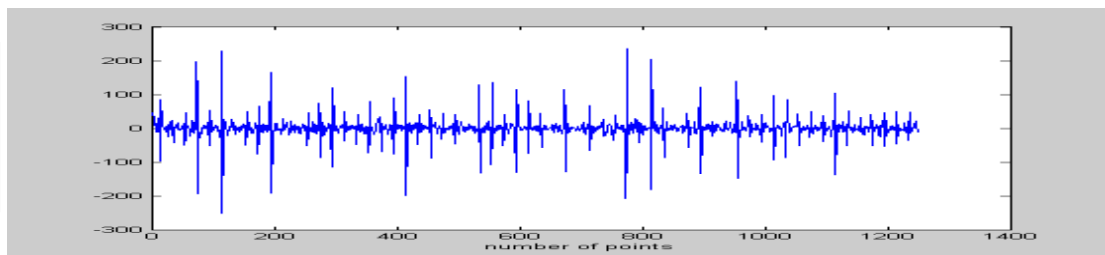


Figure (18) : Ten traces of channel two of one minute record of EEG data passed through band pass digital filter of figure (15) with($r_1=0.1$ & $r_2=0.8$)

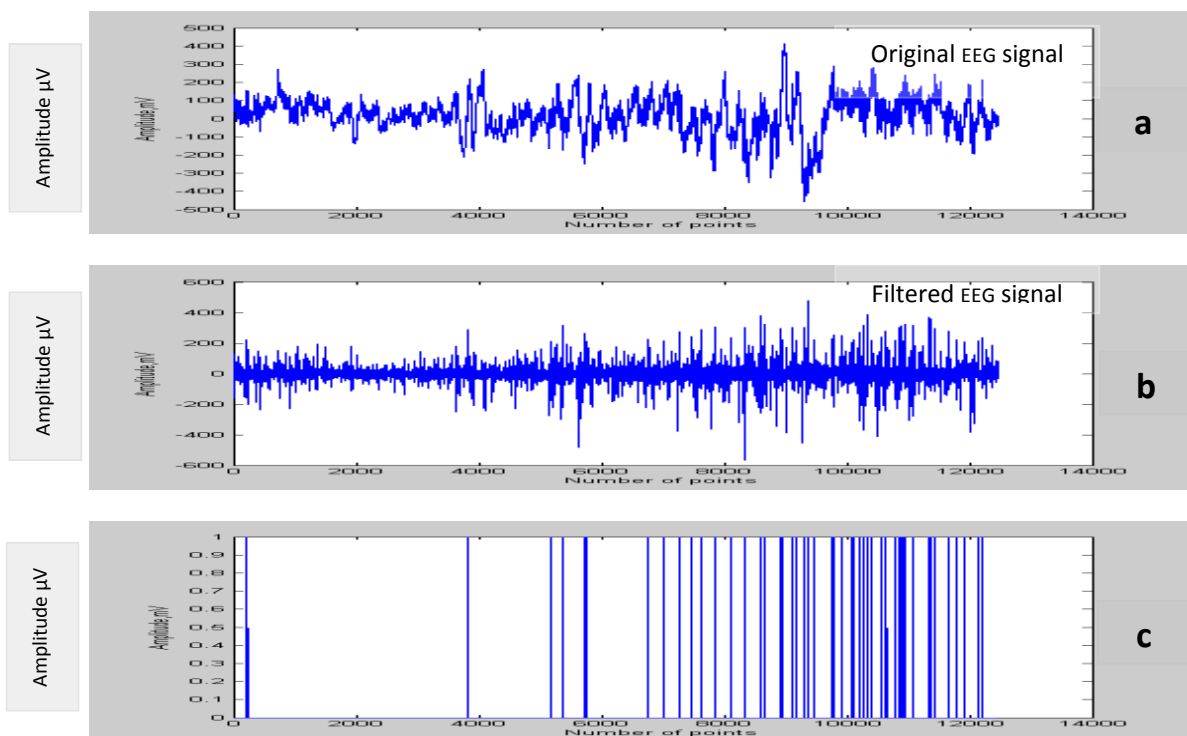
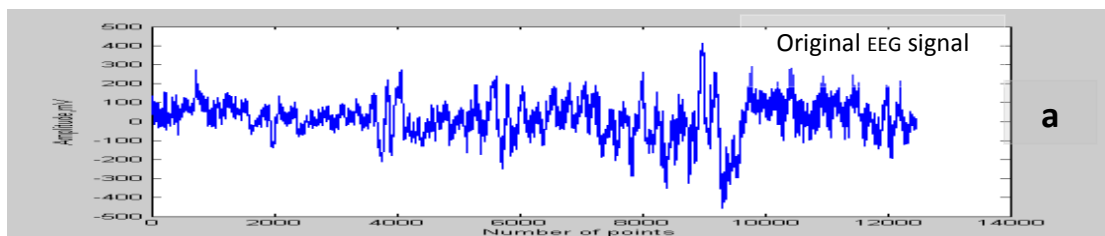


Figure (19): (a) Original EEG record,(b) the signal after being filtered with the digital filter whose z-plane is shown on figure(15), (c) the point process representation of the spike sequence for one minute record of channel two



Filtered EEG signal

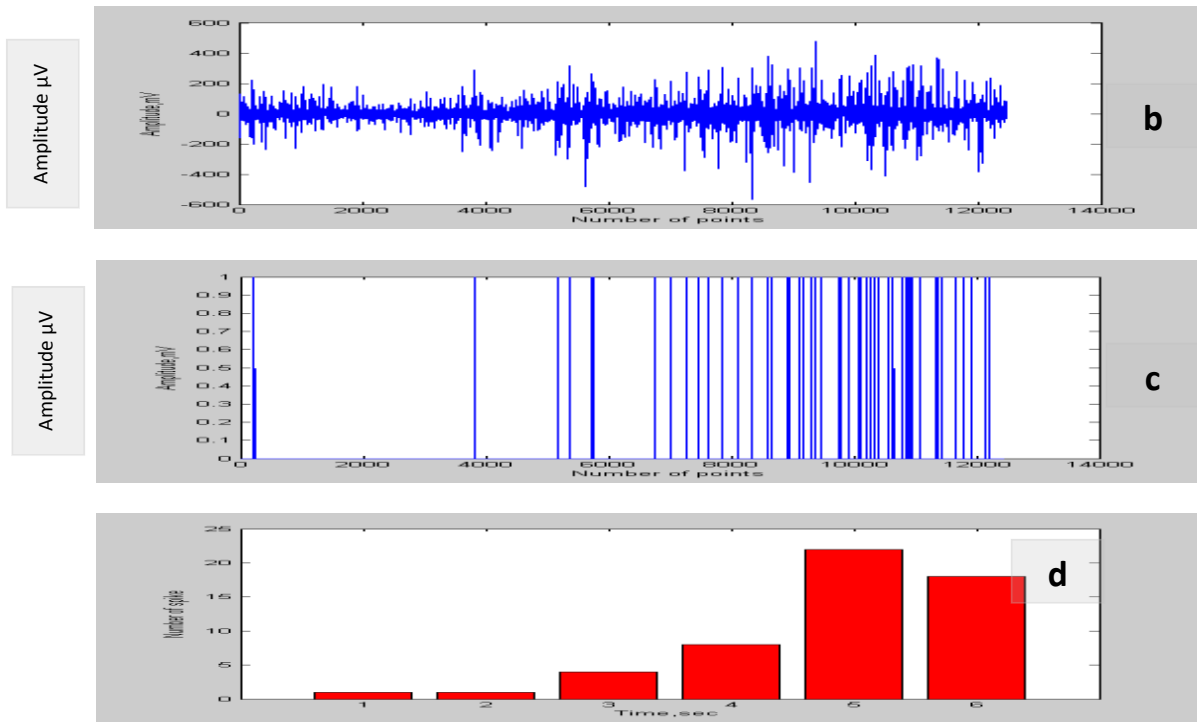


Figure (20): (a) Original record for one minute of EEG record on channel two for normal subject (b) the signal after being filtered with the digital filter whose z-plane is shown on figure (15), (c) the point process representation of the spike sequence (d) spikes per second bar chart



Figure (21): (a) Original EEG record for 6 second for epileptically patient record on channel two (b) the signal after being filtered with the digital filter whose z-plane is shown on figure (15) (c) the point process representation of the spike sequence (d) spikes per second bar chart

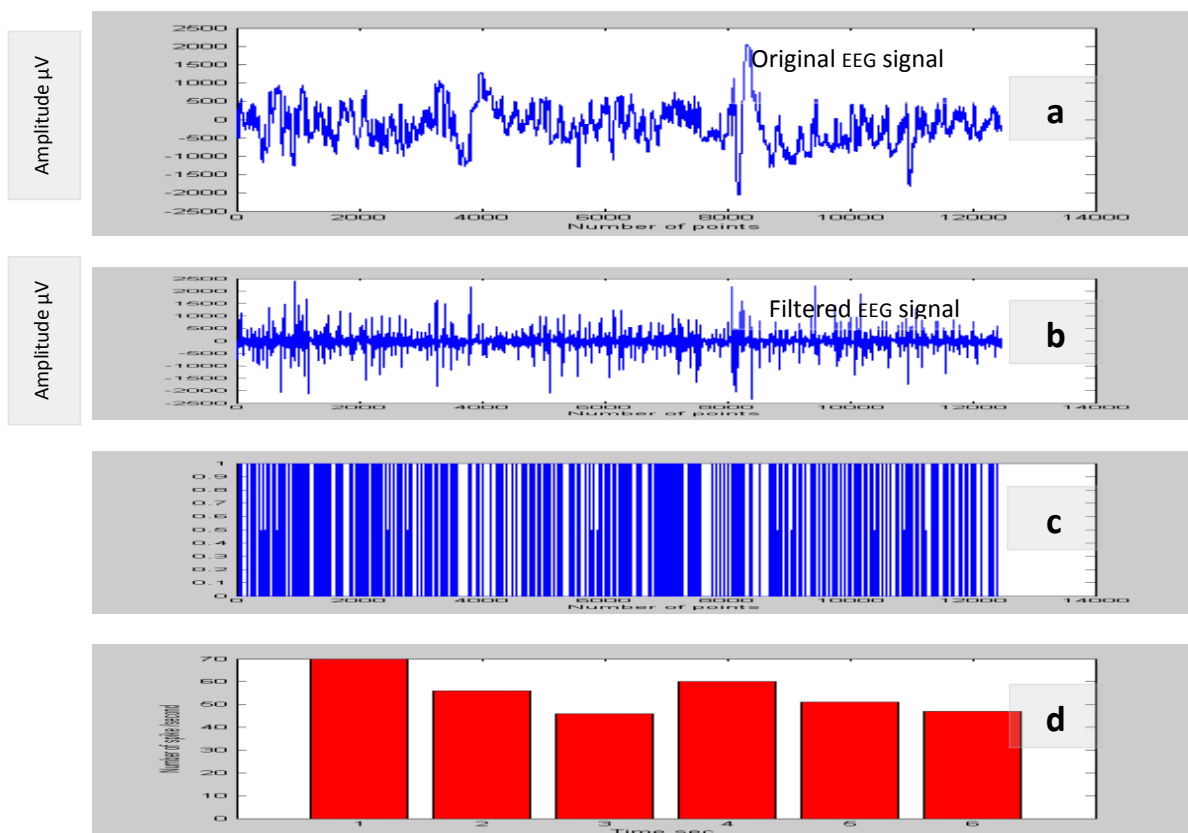


Figure (22): (a) Original EEG record for one minute for epileptically patient record on channel two (b) the signal after being filtered with the digital filter whose z-plane is shown on figure (15), (c) the point process representation of the spike sequence (d) spikes per second bar chart

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