Analysis flow of second order fluid in a vertical channel with porous wall by using the Homotopy Analysis Method (HAM)

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Abstract

In this paper, the state of non newtonian fluid of second order flow injected uniformly into the vertical channel with porous wall through one side of the channel. The equations which are used to describe the flow are the motion and the energy equations. It found that these equations are controlled by many dimensionless numbers such as Reynolds number (Re), Peclet number (Pe), Hartman number (M) and the material of fluid (α, β). The homotopy analysis method (HAM) is used to obtain the analytic solution for the velocity and heat transfer. The effect of each dimensionless parameters upon the velocity and temperature distribution is analyzed and shown graphically by using MATLAB package.

Keywords: Hartman number, Peclet number, vertical channel

المستخلص

ان هذا البحث يتضمن دراسة لجريان مائع لا نيوتيني من الرتبة الثانية في قناة عمودية ذات جدار مثقبة، المعادلات التي استخدمت لوصف حركة المائع هي معادلات الحركة ومعادلة الطاقة وقدحلت تحليلياً باستخدام طريقة الهوموتوبي حيث وجد ان هذه المعادلات تحكمها اعداد لابعدية مثل عدد رينولدز و،بلكت،هارتمان وثوابت اخرى تخص المائع . كماقمنا بدراسة تأثير تلك الاعداد اللابعدية المذكورة .وقدتم استخدام البرنامج الجاهز الماتلاب في هذا البحث .

Intrduction

Fluid is that state of matter, which capable of changing shape and capable of flowing. Fluids may be classified as "Viscous" and "Perfect" according to whether the fluid capable of exerting shearing stress or not. Viscous is called Newtonian if the relation between stress and rate of strain (state of equation) is linear, otherwise is called non – Newtonian fluid .The flow of Newtonian and non – Newtonian fluids in the porous channel has been subject extensive theoretical studies till date because many applications of them in different scientific fields. Examples, of such flow of fluid in vertical porous channel, is found in [11] as the simplest subclass for which one can hope to gain an analytic solution.

The flow of Newtonian and non – Newtonian fluids through porous channel has been investigated by numerous authors. The case of a two dimensional, incompressible, steady, laminar suction flow of Newtonian fluid in a porous channel was studied by Berman [5]. He has solved the Navier-Stokes equations by using a perturbation method for very low cross – flow Reynolds number. After his pioneering work, this problem has been studied by many researchers considering various variations in the problem [7,8].

Wang and Skalak [17] were the first persons who present the solution for a three – dimensional problem of fluid injection through one side of a long vertical channel for Newtonian fluid .They have obtained a series solution for a small value of Reyonlds number and numerical solution for small and large Reynolds number.

Huang [10] re- examined Wang and Skalak problem using a method based upon quasilinearization. Ascher [4], Sharma and Chaudhary [16] reconsidered the above –mentioned problem by introducing a second viscoelastic fluid. They obtained the second order perturbation solution by assuming that the cross– flow of Reynolds number is a small.

Baris [6] continued the last mentioned research by substituting thermodynamically compatible fluid of second grade instead of Newtonian fluid. The used analytical method by Baris was traditional perturbation solution, which was one of old analytical methods.

These scientific problems are modeled by ordinary or partial differential equations and should be solved using special techniques, because in most case, analytical solutions can't be applied to these problems. In resent years, much attention has been devoted to the newly developed methods to constant an analytical solution of these equations. One of these techniques is homotopy Analysis Method (HAM), which was introduced by Liao [14,15] and has been successfully applied to solve many types of nonlinear problems[1,2,11,12]. HAM is a powerful technique for solving linear and nonlinear partial differential equation for example the equation that appears in our problem. In most cases of nonlinear problems can be described by a set of governing linear equations with its initial / boundary conditions [14].

The paper was dependent upon work of Khalid, Ahmed [13]. They are investigated to find the velocity, heat transfer and pressure variation profiles of Newtonian fluid in a vertical channel with porous wall. The governing non- linear problems have been solved analytically by using HAM. In this paper, HAM is employed to find the velocity, heat transfer and pressure variation profiles of non- Newtonian fluid of second order in a vertical channel with porous wall and examine qualitatively the effect of non –Newtonian parameters (α,β) which are dimensionless numbers, Reynolds number Re, Hartmann number M, and Peclet number Pe on these value .

Governing Equations:

The study of second grad fluid in a vertical channel with porous wall is considered. Fig.1 shows the physical model and coordinate system. A fluid is injected through a vertical porous plate at y=D with uniform velocity U. The fluid strikes another vertical impermeable plate at y=0.It flows out through the opening the plates, due the action of gravity along the Z-axis. The distance between the walls is assumed D, is a small compared to the dimensions of plates, i.e., L $\gg B \gg D$. Due to this assumption the edge effects can be ignored and the isobars are parallel to the Z-axis.

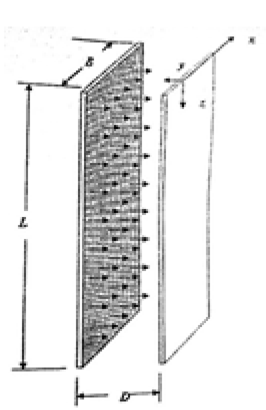


Fig.1Schesh the problem under discussions

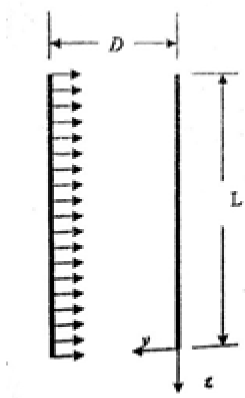


Fig.2 Fluid flow in a vertical channel

The Caushy stress tensor in such a fluid is related to the motion equations in the following manner [9].

T=-PI+μ$A_1 + \alpha_1 A_2 + \alpha_2 {A_1}^2$
(1)

where $A_1 = \nabla \boldsymbol{V} + (\nabla \boldsymbol{V})^T$

$$A_2 = \frac{dA_1}{dt} + A_1(\nabla \boldsymbol{V}) + (\nabla \boldsymbol{V})^T A_1 \qquad (2)$$

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \alpha_2 \geq 0 \qquad (3)$$

In this equation, P is the pressure, $\boldsymbol{V}$ is the velocity vector, ∇ is the gradient oparoter, $\alpha_i (i = 1,2)$ are the material moduli of fluid, d/dt is the material derivative and $A_i (i = 1,2)$ are the two first Rivilin Eriksen tensor.

Note that for α_1, α_2=0 equation (1) along with (2) descries of Newtonian fluid [13]. In addition to (1) the basic equations of the problem are in the following:

$$\nabla \boldsymbol{V} = 0 \qquad (4)$$

$$\rho(\boldsymbol{V} \nabla \boldsymbol{V}) = \nabla T + (\boldsymbol{J} \times \boldsymbol{B}) \qquad (5)$$

$$\rho Cp(\mathbf{V}\nabla T) = K\Delta T \qquad (6)$$

Equations (4), (5) and (6) are the continuity, momentum and energy equations, respectively. Where ρ is the density and $(\mathbf{J}\times \boldsymbol{B})$ is Lorenz force vector. The fluid is assumed to be steady and laminar. Substituting the stress tensor T from (1) into (5) yields:

$$\rho(\boldsymbol{V}\nabla V) = -\nabla P + \mu(\nabla^2\boldsymbol{V}) - \sigma u B^2 \qquad (7)$$

The velocity components corresponding to X ,Y ,Z direction respectively denoted by u ,v ,w, following [17] , we look for a solution, compatible with the continuity of the form :

$$u= \frac{Ux}{D} f'(\eta) \quad ,v =-Uf(\eta) \quad , \quad w=\frac{D^2 g\rho}{\mu} h(\eta) \qquad (8)$$

where $\eta = y/D$ and the prime denoted the differential with respect to η .

The boundary conditions for the velocity field are :

f(0)=0 , f(1)=1 f '(0)=0, f '(1)=0, f "(0)=6 (9)

It follows from (7) and equation of motion that :

$$\frac{\partial P}{\partial x} = \frac{Ux}{D^2}[Re(ff'' - f'^2) + f''' - Mf' + \alpha(-ff'''' + 2f'f''' + 3f''^2) + \beta(2f''^2)] \qquad (10)$$

$$\frac{\partial P}{\partial \eta} = -Reff' - \frac{\mu U}{D}[f'' + \alpha\left(ff''' + 6f'f'' + \frac{8x^2}{D^2} f''f'''\right) + \beta\left(8f'f'' + \frac{2x^2}{D^2} f''f'''\right)] \qquad (11)$$

Where the cross –flow Reynolds number ,Re, M is the Hartmann number, and α,β are the dimensionless numbers ,are defined through respectively.

$$Re= \frac{\rho UD}{\mu} \quad , \quad M= \frac{\sigma uB}{\mu} \quad , \quad \alpha = \frac{U\alpha_1}{\mu D} \quad , \quad \beta= \frac{U\alpha_2}{\mu D} \qquad (12)$$

Integrating (11) with respect to η ,obtained equation is:

$$P(x,\eta) = -\frac{1}{2} f^2 + \frac{\mu U}{D}\left[-f' + \alpha\left(ff'' + \frac{5}{2} f'^2 + \frac{4x^2}{D^2} f''^2\right) + \beta\left(\frac{x^2}{D^2} f''^2 + 4f'^2\right)\right] + \varphi(x) \qquad (13)$$

Where $\varphi(x)$ is arbitrary function of x. different ion of the above equation with respect to x yields:

$$\frac{\partial P}{\partial x} = \frac{\mu U}{D}\left[\frac{8x}{D^2}\alpha f''^2 + \frac{2x}{D^2}\beta f''^2\right] + \frac{d\varphi}{dx} \qquad (14)$$

Combining of (14) and (10)

$\frac{d\varphi}{dx} = \frac{\mu Ux}{D^3}[Re(ff'' - f'^2) + f''' - Mf' + \alpha(-ff'''' + 2f'f''' + 3f''^2) + \beta(2f''^2)] - \frac{\mu U}{D}\left[\frac{8x}{D^2}\alpha f''^2 + \frac{2x}{D^2}\beta f''^2\right]$ (15)

And

$\frac{D^3}{\mu Ux}\frac{d\varphi}{dx} = [Re(ff'' - f'^2) + f''' - Mf' + \alpha(-ff'''' + 2f'f''' - 5f''^2)$ (16)

It is apparent that the quantity in parentheses in (16) must be independent of η.Hence ,the following equation for f is:

$f''' + Re(ff'' - f'^2) - Mf' + \alpha(-ff'''' + 2f'f''' - 5f''^2) = C$ (17)

Where C arbitrary constant which takes value

$C = f'''(0)$ (18)

Now differentiating (17) with respect to η yields :

$f'''' + Re(f'''f - f'f'') - Mf'' + \alpha(-ff''''' + f'f'''' - 8f''f''')$ (19)

by using (17) ,$\varphi(x)$ can be written as

$\varphi(x) = \frac{\mu Ux^2}{2D^3}C + C_0$ (20)

Where C_0 is the constant of integration. Inserting $\varphi(x)$ from (20) into (13) :

$P(x,\eta) = -\frac{1}{2}f^2 + \frac{\mu U}{D}\left[-f' + \alpha\left(ff'' + \frac{5}{2}f'^2 + \frac{4x^2}{D^2}f''^2\right) + \beta\left(\frac{x^2}{D^2}f''^2 + 4f'^2\right)\right] + \frac{\mu Ux^2}{2D^3}C + C_0$
(21)

from (21) ,the pressure variation in x and y direction can be written in dimensional form as follows:

$P(x) = \frac{P(0,\eta) - p(x,\eta)}{\rho U^2} = -\frac{1}{Re}(4\alpha f''^2 + \beta f''^2 + \frac{1}{2}f''''(0))(\frac{x}{D})^2$ (22)

$P(y) = \frac{P(x,0) - p(x,\eta)}{\rho U^2} = \frac{f^2}{2} + \frac{1}{Re}(f' + \alpha(ff'' + \frac{5}{2}f'^2) + 4\beta f'^2)$ (23)

Note that the equations (10),(11),(13),(15),(16),(17),(19) and (21) becomes in Newtonian flow [13] where we put α and $\beta = 0$.

Equations for Temperature 3- Governing

In this section ,temperature field as below

$T = T_0 + (T_1 - T_0)\theta(\eta)$ (24)

where T_0, T_1 are the temperatures of impermeable and porous plates, respectively and with constant value . Substituting(8)and(24)into (6) lead to the following equation:

$$\theta'' + Pe\theta' = 0 \qquad (25)$$

where Pe $=\rho UDcp/$k is the Peclet number .Equation (25) is solved subject to the boundary conditions

$$\theta(0) = 0 , \quad \theta(1) = 1 \qquad (26)$$

4- Solution Using Homotopy Analysis Method

In this section HAM is applied to solve (19) subject to the boundary conditions (9) .The initial guesses and linear operators are chosen in the following :

$$f_0(\eta) = 3\eta^2 - 2\eta^3 \qquad (27)$$

As the initial guess approximation for $f(\eta)$ is

$$L_1(f) = f^v \qquad (28)$$

As the auxiliary linear operator has the property:

$$L(c_1 + c_2\eta + c_3\eta^2 + c_4\eta^3 + c_5\eta^5) = 0 \qquad (29)$$

And $c_i(i = 1 - 5)$ are constant. Let p∈ [0,1]denotes the embedding parameter and h indicates non zero auxiliary parameters .Then the following equation are constructed:

$$(1 - p)L_1(f(\eta; p) - f_0(\ \eta)] = ph_1N_1[f(\eta; p)] \qquad (30)$$

$$f(0;p) = 0, f'(0; p) = 0 \ , f(1; p) = 1, f'(1; p) = 1 \qquad (31)$$

$$N_1[f(\eta; p)] = f''''(\eta; p) + Re\big(f'''(\eta; p)f(\eta; p) - f'(\eta; p)f''(\eta; p)\big) - Mf''(\eta; p) + \alpha(-f(\eta; p)f'''''(\eta; p) + f'(\eta; p)f''''(\eta; p) - 8f''(\eta; p)f'''(\eta; p)) = 0 \qquad (32)$$

for p=0 and p=1:

$$f(\eta; p) = f_0(\eta), \quad f(\eta;1) = f(\eta) \qquad (33)$$

When p increases from 0 to 1 then $f(\eta; p)$vary form $f_0(\eta)$ to $f(\eta)$.By using Taylor's theorem and using (33):

$$f(\eta; p) = f_0(\eta)+\sum_{m-1}^{\infty} f_m(\eta)p^m, f_m(\eta) = \frac{1}{m!}\frac{\partial^m(f(\eta;p))}{\partial p^m} \qquad (34)$$

$$f(\eta) = f_0(\eta) + \sum_{m-1}^{\infty} f_m(\eta) \qquad (35)$$

mth − order deformation equations are: The

$$L[(f_m(\eta) - X_m f_{m-1}(\eta)] = hR^f{}_m(\eta), \quad (36)$$

The boundary conditions are:

$$f_m(0) = f'_m(0) = f_m(1) = f'_m(1) = 0 \quad , f''_m(0) = 6 \quad (37)$$

Where $R^f{}_m(\eta) = f''''+\text{Re}\sum_{i=0}^{m-1}(f_{m-1}f'''_i - f'_{m-1}f''_i) - Mf''_{m-1} + \alpha(\sum_{i=0}^{m-1}(-f_{m-1}f'''''_i + f'_{m-1}f''''_i - 8f''_{m-1}f'''_i))$ (38)

$$X_m = \begin{cases} 0 & m \le 1 \\ 1 & m > 1 \end{cases} \quad (39)$$

To find the solution of m th -order deformation ,we shall use the symbolic software MATLAB up to first few order of approximation . We found the solution up to 3 th order approximation and they are:

$f_1=$

$24Reh\eta^3 - \eta^4((Mh)/4 - 24\alpha h) - \eta^5((Mh/20)-(24\alpha h)/5) - \eta^3(Mh - 96\alpha h) - \eta^2(3Mh - 288\alpha h) - \eta^4((3Reh)/2 - (Mh)/2 + 48\alpha h) - \eta^5((3Reh/10 - (Mh)/10 + (48\alpha h)/5) - \eta^6(Reh)/20 - (Mh)/60 + (8\alpha h)/5) - \eta^3(6Reh - 2Mh + 129\alpha h) - \eta^2(18Reh - 6Mh + 576\alpha h) - \eta(6Mh - 576\alpha h) - 6Reh\eta^4 - (6Reh\eta^5)/5 - (Reh\eta^6)/5 - (Reh\eta^7)/35 - (Reh\eta^8)/140$

$f_2=$
$(37Re^2h^2\eta^9)/210 - \eta^8((6Re^2h^2)/35 - (MReh^2)/140 + (183Re\alpha h^2)/35) - \eta^9((Re^2h^2)/28 - (MReh^2)/504 + (67Re\alpha h^2)/210) - \eta^6(18Re^2h^2 - MReh^2 + (804Re\alpha h^2)/5) - \eta^7((18Re^2h^2)/7 - (MReh^2)/7 + (804Re\alpha h^2)/35)\ldots\ldots\ldots\ldots\ldots.$

$f_3 =$

$\eta^{16}((181Re^3h^3)/100900800 + (MRe^2h^3)/42042000 + (11Re^2\alpha h^3)/1092000) - \eta^8((9Re^2h^2)/28 - (MReh^2)/56 + (201Re\alpha h^2)/70) - \eta^8((6Re^2h^2)/35 - (MReh^2)/140 + (1831Re\alpha h^2)/35) - \eta^9((Re^2h^2)/28 - (MReh^2)/504 + (67Re\alpha h^2)/210) - \ldots\ldots\ldots$

5- Convergence of Solution (4)

We notice that the explicit analytical expression in eq.(34) contain the auxiliary parameter h_1.As pointed out by Liao [14] ,the convergence region and the rate of approximations given by the HAM are strongly depending on h_1.. By means of so-called h-curve for the velocity profile figure (3). The range of admissible value of $h_1$for the velocity profile when Re=10, M=1, and $\alpha = 1$ is $-1.8 \ge h_1 \ge -0.2$. Note that if $\alpha = 0$, then the series belong in [13].

6-Solution of Energy Equation

In this section HAM is applied to solve (25) subject to the boundary conditions (26) The initial guesses and linear operators are chosen in the following:

$$\theta(\eta) = \eta \quad (40)$$

As the initial guess approximation for $\theta(\eta)$ is

$L_2(\theta) = \theta''$ (41)

As the auxiliary linear operator has the property:

$L(c_1 + c_2\eta) = 0$ (42)

And $c_i(i = 1 - 2)$ are constant. Let p∈ [0,1]denotes the embedding parameter and h indicates non zero auxiliary parameters .Then the following equation are constructed:

Zeroth − order deformation equations

$(1 - p)L_1(\theta(\eta; p) - \theta_0(\ \eta)] = ph_2N_2[\theta(\eta; p)]$ (43)

$\theta(0;p) = 0, \qquad \theta(1; p) = 1$ (44)

$N_2[\theta(\eta; p)] = \theta''(\eta; p) + Pe\big(f(\eta; p)\theta'(\eta; p)\big)=0$ (45)

for p=0 and p=1:

$\theta(\eta; 0) = \theta_0(\eta), \theta(\eta;1) = \theta(\eta)$ (46)

When p increases from 0 to 1 then $\theta(\eta; p)$vary form $\theta_0(\eta)$ to $\theta(\eta)$.By using Taylor's theorem and using (46):

$\theta(\eta; p) = \theta_0(\eta) + \sum_{m-1}^{\infty} \theta_m(\eta)p^m, \ \theta_m(\eta) = \frac{1}{m!}\frac{\partial^m(\theta(\eta;p))}{\partial p^m}$ (47)

$\theta(\eta) = \theta_0(\eta) + \sum_{m-1}^{\infty} \theta_m(\eta)$ (48)

The mth − order deformation equations

$\text{L}[(\theta_m(\eta) - X_m\theta_{m-1}(\eta)] = hR^{\theta}{}_m(\eta),$ (49) The boundary conditions are:

$\theta_m(0) = \theta_m(1) = 0$ (50)

Where $R^{\theta}{}_m(\eta) = \theta''{}_{m-1} + \text{Pe}\sum_{i=0}^{m-1}(f_{m-1}\theta'{}_i)$ (51)

$X_m = \begin{cases} 0 & m \leq 1 \\ 1 & m > 1 \end{cases}$ (52)

To find the solution of m th -order deformation ,we shall use the symbolic software MATLAB up to first few order of approximation . we found the solution up to 3 rd. order approximation and they are:

$\theta_1 =$

$-(Peh\eta^4(2\eta - 5))/20 - (Peh\eta^3(\eta - 2))/2$

$\theta_2 =$

$\eta^8((Pe^2h^2)/8 - (RePeh^2)/280 - \eta^2(3MPeh^2 - 288Pe\alpha h^2) - \eta^3((MPeh^2 - 96Pe\alpha h^2) + \eta^9((Pe^2h^2)/72 - (RePeh^2)/2520) + \eta^5((9Pe^2h^2)/5 + (MPeh^2)/20 - (3PeReh^2)/2 - (24Pe\alpha h^2)/5) - \eta^6((3Pe^2h^2)/120 - (MPeh^2)/20 + (PeReh^2)/4 + (4Pe\alpha h^2)/5) + \eta^6((3Pe^2h^2)/10 + (MPeh^2)/120 - (PeReh^2)/4 - (4Pe\alpha h^2)/5) - \cdots$ ……………………

$\theta_3 =$

$\eta^8((Pe^2h^2)/8 - (RePeh^2)/280 - \eta^2((3MPeh^2 - 288\alpha Peh^2) - \eta^2(3MPeh^3 - 288Pe\alpha h^3) - \eta^3(MPeh^2 - 96Pe\alpha h^2) + \eta^9((Pe^2h^2)/72 - (RePeh^2)/2520) - \eta^7((M^2Peh^3)/168 + (17MPe^2h^3)/28 - (5MPeReh^3)/14 - (17MPe\alpha h^3)/7) - \cdots$…………………

7-Convergence of Solution (6)

The explicit analytical expression in eq.(47) contain the auxiliary parameter h_2.As pointed out by Liao [14] ,the convergence region and the rate of approximations given by the HAM are strongly depending on h_2.. By means of so-called h-curve for the temperature profile figure (4). The range of admissible value of h_2 for the temperature profile when Re=10, M=1, and $\alpha = 1$ is $-1.8 \geq h_2 \geq -0.2$,note that if $\alpha = 0$, then the series belong in [13].

8-Results analysis

In this section, we studied the effect of dimensionless parameters that governing the momentum and energy equations, upon the normal, tangential velocities and temperature of vesicant fluid of second order in a vertical channel. All results are plotted by MATLAB package. Figures (5,6) shows the effect of Hartmann number M on the normal and the tangential velocity components (η), we keep Re=10,$\alpha = 1$ and M has been given 0.001,1,5 and 15. The following results are observed: As Hartmann M increases, there is a small decreasing in normal and tangential velocity component range. Figure 7 shows the function which correspond to the velocity components have been plotted versus η for Re=1,10,20 fixed M=1,and $\alpha = 1$. For increase values of Reynolds number then the velocity increasing.

Figure 9 illustrates the effect of dimensionless parameter α on the normal velocity profiles for fixed Re=10,M=1 and α=0.002,2,4.5.It obvious from this this figure that the effect of α is very strong on the normal velocity profile where it increases, and if we put α =0,then the velocity profile becomes in flow of Newtonian fluid cases[13].

Figure 10 depicts the tangential velocity for Re=10,M=1 and $\alpha = 1,3,4.5$. It is obvious for this figure that the tangential velocity increases if the value of α is large.

Figures 11 and 12, the effects of section and injection have been investigated, as it showed by contours.

Figure 15 depicts the profiles of temperature in viscoelastic fluid in a vertical channel with porous wall .when Re=10,M=1, α =1 that the effect of Peclet number on temperature profile was shown. According to definition of Peclet number was increased of uniform injection velocity. Due the convection effects, increasing of Peclet number led to intensity the temperature distribution in the channel. Figure 13 and 14 present the pressure variations in x and y directions, respectively when Re=10,M=1. From the evident the pressure variations will increase with increasing of non - Newtonian parameters α andβ.

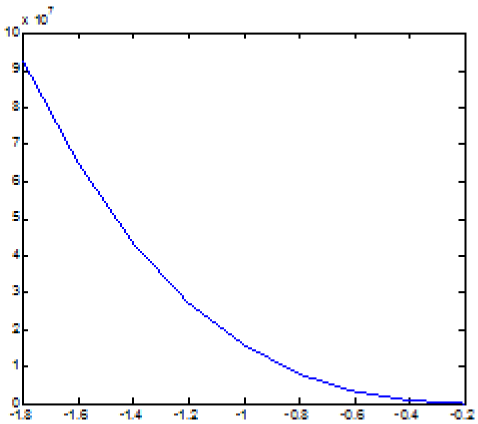


Fig.3 The h_1 curve for Re=10,M=1,α=1

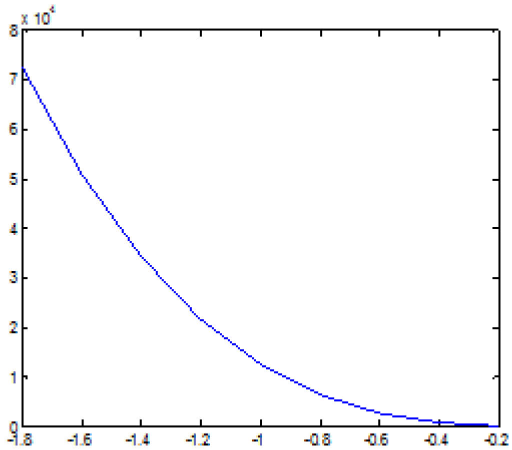


Fig.4 The h_2 curve for Re=10,M=1,α=1,Pe=1

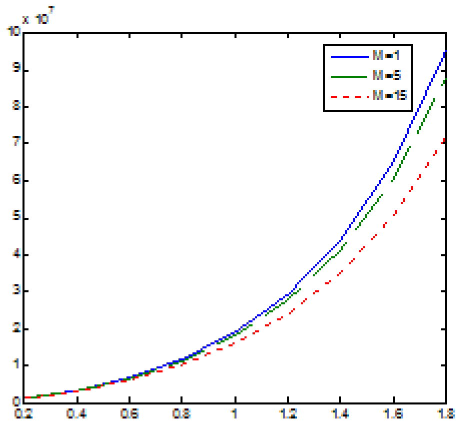


Fig.5 The velocity for Re=10,$\alpha = 1$

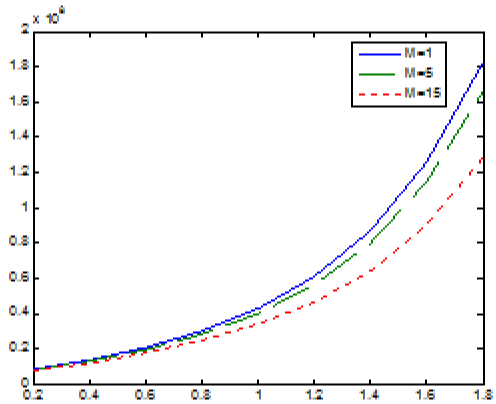


Fig.6 The tangential velocity for Re=10,$\alpha = 1$

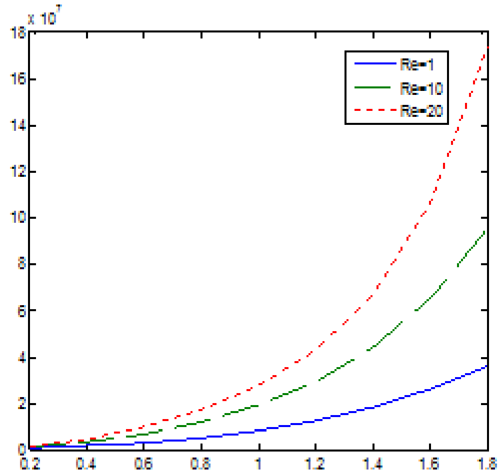


Fig7 The velocity for M=1, $\alpha = 1$

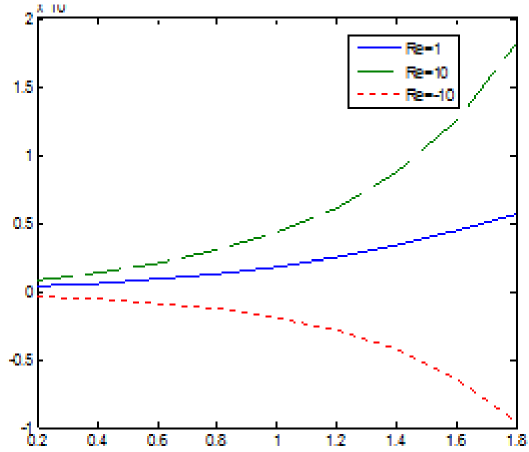


Fig.8 The tangential velocity for M=1, $\alpha = 1$

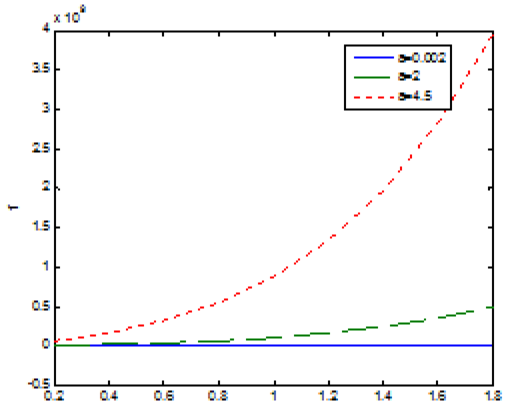


Fig.9 The velocity for M=1, $Re = 10$

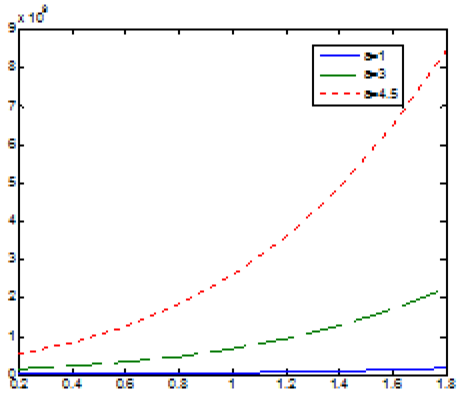
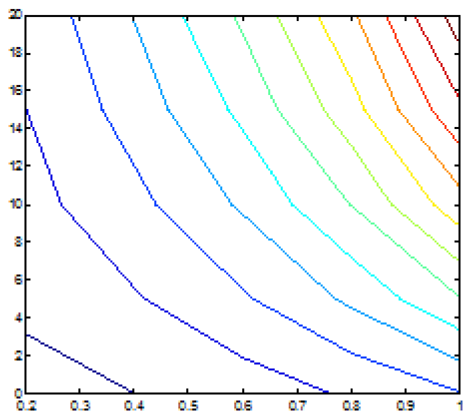


Fig10 The tangential al velocity for M=1, Re=10



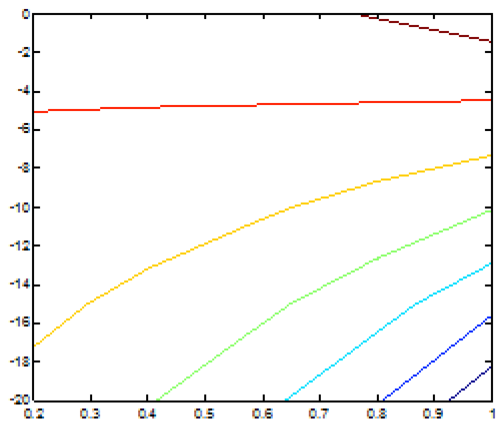
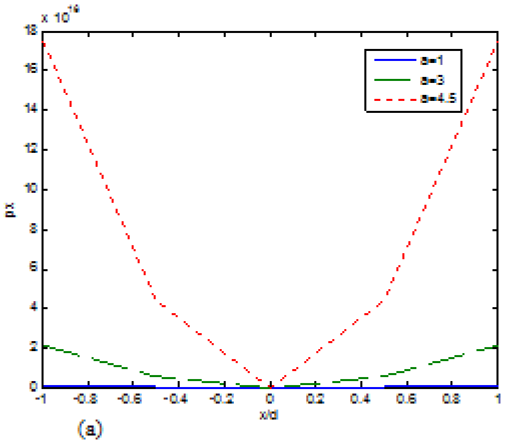
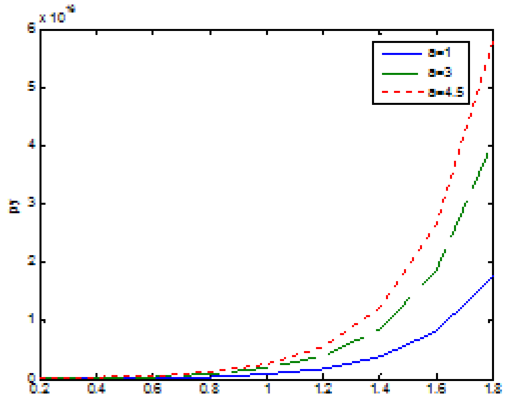


Fig.11The tangential velocity contour for, $Re = 10$ Fig.12 The tangential velocity contour for, $Re = -10$

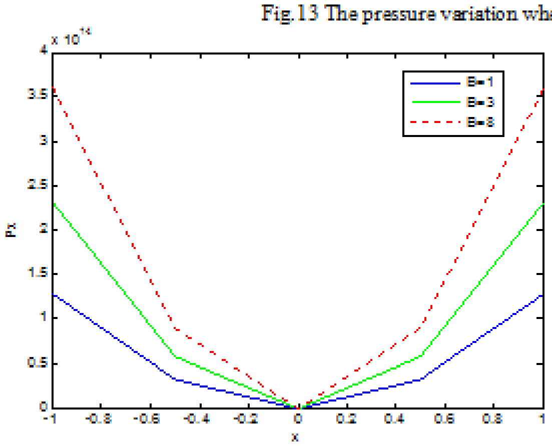


(a)

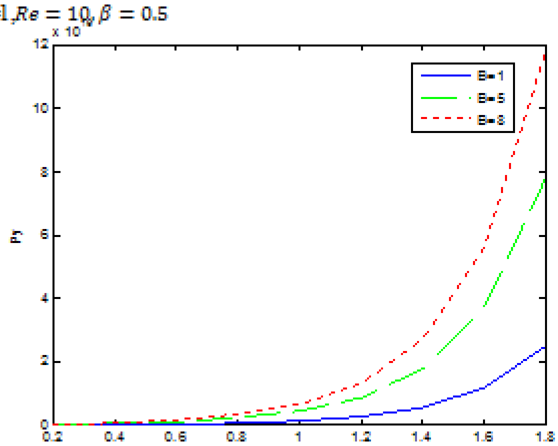


(b)

Fig.13 The pressure variation when M=1, $Re = 10, \beta = 0.5$



(a)



(b)

Fig.14 The pressure variation when M=1, $Re = 10, \alpha = 1$

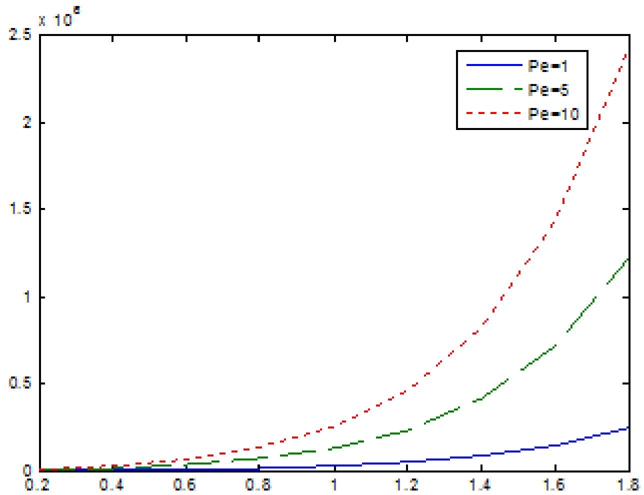


Fig.15The temperature variation when M=1, $Re = 10, \alpha = 1$

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