Analysis of Magneto Hydrodynamic of Second Order Fluid Flow in a Micro-Channel and heat Transfer between Two Parallel Plates

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Abstract

In present study, analysis of magnetic field was studied in the state of non Newtonian fluid of second order flows and heat transfer in micro channel between two parallel plates, introduced. The equations are used to describe the flow are the motion and the energy equations. It found that these equations are controlled by many dimensionless numbers such as Reynolds number (Re), magnetic field parameter (M), physical quantity at wall (W), Knudsen number (Kn), Peclet number (Pe), Brinkman number (Br) and the material of fluid (α, β). The homotopy analysis method (HAM) is used to obtain the analytic solution for the velocity and heat transfer, the effect of each dimensionless parameters upon the velocity and heat distribution is analyzed and shown graphically by using MATLAB package.

Key words; Second order fluid, The velocity profile, The heat transfer.

تحليلالمغناطيسيةهيدروديناميكية لتدفق السائل منالدرجةالثانيةفيقناةالصغرى،ونقلالحرارةبيناثنينملوحاتالموازية

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المستخلص

في هذا البحث دراسة الحقل المغناطيسي لجريان مائع لا نيوتيني من الرتبة الثانية وانتقال الحرارة في الانابيب الدقيقة بين صفيحتين متوازيتين, المعادلات التي استخدمت لوصف حركة المائع هي معادلات الحركة ومعادلة الطاقة وقدحلت تحليلياً باستخدام طريقة الهوموتوبي حيث وجد ان هذه المعادلات تحكمها اعداد لابعدية مثل عدد رينولدز،بلكت،هارتمان وثوابت اخرى تخص المائع. وقمنا بدراسة تأثير تلك الاعداد اللابعدية المذكورة.وقدتمت هذه الدراسة باستخدام البرنامج الجاهز الماتلاب.

الكلمات المفتاحية:سائل الدرجة الثانية, توصيف السرعة, نقل الحرارة

Introduction

Magneto-fluid dynamics (MHD) is that branch of applied mathematics, which deals with the flow of electrically conducting fluids in electric and magnetic fields. It unified in a common framework the electromagnetic and fluid-dynamic theories to yield a description of the concurrent effects of the magnetic field on the flow and the flow on the magnetic field.

In view of the abundant applications of non-Newtonian fluids in industry and technology, the interest in the study of such fluids has been increased during the last few years. Mathematicians and computer scientist have been involved in carrying out flow analyses of the non-Newtonian fluids in various aspects. Several constitutive expressions for these fluids have been suggested. These equations differ between the shear stress and rate of strain in view of the different characteristics of the non-Newtonian fluids. As a consequence of these constitutive equations, the resulting equations for non-Newtonian fluids in general are more complicated and of high order in comparison to the Navier- Stokes equations.
Considerable efforts have been devoted to studying the non-Newtonian fluids through analytic and numerical treatments. Some progress on the topic can be mentioned: in the studies [2, 11, 14-16]. In all of these studies, constant viscosity fluids (Newtonian fluids) are used.
A systematic research on micro devices started in the late 1980's.Micro ducts, micro nozzles, micro pumps, micro turbines and microvalves are the examples of the devices involving liquid and gas flows.

Modeling mass, momentum and energy transport may be necessary. Slip, rarefaction, compressibility, intermolecular forces andother unconventional effects. The Knudsen number (Kn) can classify thegas flow in micro channel into four flow regimes: continuum flow (Kn<0.001), slip flow (0.001 <Kn< 0.1), transition flow (0.1 <Kn< 10) and free molecular flow (Kn> 10) [5]. Since Navier–Stokes (N–S) equations are not valid for Kn beyond 0.1, the lattice Boltzmann method (LBM)
was developed as an alternative numerical scheme [23] and [19].However, for flows in continuum and slip regimes, Eckert and Drake [6] have indicated that there is strong evidence to use the N–S equations modified by boundary conditions. Tsien [20] originally designated theregime next to continuum flow as the "slip flow", following Maxwell and Smoluchowski in assuming that the first failure of continuum theory would occur at gas–solid interfaces, where the empirical conditions of continuity of tangential velocity and temperature should give way to the slip and temperature-jump boundary conditions. Studies of the continuum theory warn that in principle the N–S-plus-slip theory lacks internal consistency, but the try-it-and-see approach has yielded a substantial body of practically satisfactory results[19]and Liu [12] .
The Homotopy Analysis Method (HAM) is a powerful technique for solving linear and nonlinear partial differential equation ,for example the equation that appears in our problem . In most cases of nonlinear problems can be described by a set of governing linear equations with its initial / boundary conditions.[12].

The main paper that upon, is the work of Marwan, Ahmed. [17] ,they are studied of MHD on flow of Newtonian fluid and heat transfer between two plates .The governing non- linear problems have been solved analytically by using (HAM).

In this study HAM is employed to find the velocity, heat transfer of non- Newtonian fluid of second order by assumption:

1. Steady flow of incompressible fluid.
2. Two -dimensional and laminar fluid flow.
3. Constant fluid properties i,eC_p, k,μall remain constants.

4. Only conductive and convective energies in the flow are considered.
5. Heat generation on account of fluid friction (*known as viscous work*) being small (as the flow velocities are moderate). Finally, the results and discussions are given the effects of the various parameters of interest for the velocity and heat transfer.

2- Governing Equations:

Let (x, y, t) denote the Cartesian coordinates, V=(u, v) is the velocity vector in these directions, and t is the time. As depicted in Fig 1, the inlet velocity and temperature are assumed to be uniform, the distance between the two parallel plates is 2d. The governing equations based on the Navier-stokes Equations with slip-flow boundary conditions. The process is assumed tobe two-dimensional steady (all derivatives w.r.t time are zero) laminar flow and the non- Newtonian fluid of second order. The body forces and the effect of compressibility are neglected and MHD on flow and heat transfer in micro-channels between two parallel

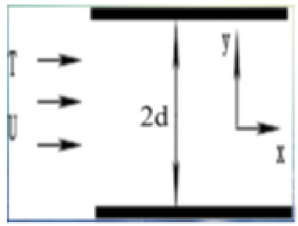

Figure 1. Microchannel between two parallel plates

The Caushy stress tensor in such a fluid is related to the motion equations in the following manner [7].

$$T=-PI+\mu A_1 + \alpha_1 A_2 + \alpha_2 {A_1}^2 \qquad (1)$$

where$A_1 = \nabla \boldsymbol{V} + (\nabla \boldsymbol{V})^T$

$$A_2 = \frac{dA_1}{dt} + A_1(\nabla \boldsymbol{V}) + (\nabla \boldsymbol{V})^T A_1 \qquad (2)$$

$$\mu \geq 0,\ \alpha_1 \geq 0, \alpha_2 \geq 0 \qquad (3)$$

In this equation, P is the pressure, $\boldsymbol{V}$is the velocity vector, ∇ is the gradient oparoter, $\alpha_i(i = 1,2)$ are the material moduli of fluid ,d/dt is the material derivative, and $A_i(i = 1,2)$are the two first Rivilin Eriksen tensor .

Note that for α_1, α_2=0 equation (1) along with (2) descries of Newtonian fluid [17].

In addition to (1) the basic equations of the problem are in the following:

$$\nabla \boldsymbol{V} = 0 \qquad (4)$$

$$(\nabla \boldsymbol{V}) = \nabla T + (\boldsymbol{J} \times \boldsymbol{B}) \qquad (5)$$

$$\rho Cp(\mathbf{V}\nabla T) = K\Delta T (6)$$

Equations (4),(5) and (6) are the continuity, momentum ,and energy equations respectively .Where ρ is the density and ($\mathbf{J}\times \boldsymbol{B}$)is Lorenz force vector .The fluid is assumed to be steady and laminar .substituting the stress tensor T from (1) into (5)yields:

$$\rho(\boldsymbol{V}\nabla V) = -\nabla P + \mu(\nabla^2 \boldsymbol{V})\sigma u B^2 \qquad (7)$$

The velocity components corresponding to X ,Y direction respectively denoted by u ,v, following [15] , compatible with the continuity of the form :

$$u= \frac{Ux}{H} f'(\eta), v = -Uf(\eta) \quad (8)$$

where$\eta = y/H$ and the prime denoted the differential with respect to η

The boundary conditions for the velocity field are :

$$f''(0) - Knf'(0) = 0 \quad , \quad f''(0) = -1 \quad , f''(10 = 0 \quad (9)$$

It follows from (7) and equation of motion that :

$$\frac{\partial P}{\partial x} = \frac{Ux}{H^2}[Re(ff'' - f'^2) + f''' - Mf' + \alpha\left(-ff'''' + 2f'f''' + 3f''^2\right) + \beta(2f''^2)] \quad (10)$$

$$\frac{\partial P}{\partial y} = -Reff' - \frac{\mu U}{H}[f'' + \alpha\left(ff''' + 6f'f'' + \frac{8x^2}{H^2}f''f'''\right) + \beta\left(8f'f'' + \frac{2x^2}{H^2}f''f'''\right)] \quad (11)$$

Where the cross –flow Reynolds number ,Re, M is the Hartmann number(MHD) number, and α,β are the dimensionless numbers ,are defined through respectively.

$$Re=\frac{\rho UH}{\mu} \quad , \quad M=\frac{\sigma uB^2}{\mu} \quad , \quad \alpha = \frac{U\alpha_1}{\mu H}, \quad \beta=\frac{U\alpha_2}{\mu H} \quad (12)$$

The derivative of equation(10) w.r.t y gives

$$\frac{\partial}{\partial y}\left(\frac{\partial P}{\partial x}\right) = 0 \quad (13)$$

It can be concluded from the last equation that the function $\frac{\partial P}{\partial x}$ isindependent of variable y, which means we can assume:

$$\frac{\partial P}{\partial x} = C_x \quad (14)$$

WhereC_x is a constant

By using equation (14) into (10)

$$\frac{H^3}{\nu U x} C_x = [Re(ff'' - f'^2) + f''' - Mf' + \alpha\left(-ff'''' + 2f'f''' + 3f''^2\right) + \beta(2f''^2)] \tag{15}$$

It is apparent that the quantity in parentheses in (15) must be independent of η.Hence ,the following equation for f is:

$$[Re(ff'' - f'^2) + f''' + W - Mf' + \alpha\left(-ff'''' + 2f'f''' + 3f''^2\right) + \beta(2f''^2)] = 0 \tag{16}$$

Where $W = \frac{H^3}{\nu U x} C_x$is the physical quantity atwall

Note that the equations (11),(12),(13),(15) and (16) becomes in Newtonian flow [17] where we put α and $\beta = 0$.

Equations for Temperature3- Governing

In this section ,temperature field as below

$$\theta(\eta) = \frac{H}{x^2} \frac{(T_1 - T_0)}{(T - T_\circ)} \tag{17}$$

whereT_0, T_1 are the temperatures and with constant value. Substituting (8) and (17) into (6) lead to the following equation:

$$\theta'' + Brf''^2 - 2Pef'\theta + Pef\theta' = 0 \tag{18}$$

Where Br$=\frac{1}{\lambda} \frac{\mu U^2}{T_1 - T_0}$ Pe $=\rho U H cp/$k is the Peclet number .Equation (18) is solved subject to the boundary conditions

$$\theta(0) - Kn\theta(0) = 0\,, \quad \theta''(1) = 1 \tag{19}$$

4- Solution Using Homotopy Analysis Method

In this section HAM is applied to solve (16) subject to the boundary conditions (9) .The initial guesses and linear operators are chosen in the following :

$$f_0(\eta) = \frac{1}{6}\eta^3 - \frac{1}{2}\eta^2 - Kn\eta \quad (20)$$

As the initial guess approximation for $f(\eta)$is

$L_1(f) = f''''$(21)

As the auxiliary linear operator has the property:

$L(c_1 + c_2\eta + c_3\eta^2 + c_4\eta^3) = 0$(22)

And$c_i(i = 1 - 4)$are constant. Let p∈ [0,1]denotes the embedding parameter and h indicates non zero auxiliary parameters .Then the following equation are constructed:

$(1 - p)L_1(f(\eta;p) - f_0(\eta)] = ph_1N_1[f(\eta;p)]$(23)

f'(0;p) –Kn $f''(0;p) = 0$, $f(0;p) = 0, f''(0;p) = 0, f''(1;p) = 1$

(24)

$$N_1[f(\eta;p)] = f'''(\eta;p) + Re\big(f'(\eta;p)f(\eta;p) - f'(\eta;p)f'(\eta;p)\big) - Mf'(\eta;p) + W + \alpha\big(-f(\eta;p)f''''(\eta;p) + 2f'(\eta;p)f'''(\eta;p) + 3f''(\eta;p)f''(\eta;p)\big) + \beta 2(f''(\eta;p)f''(\eta;p)) = 0$$

(25)

for p=0 and p=1:

$f(\eta;p) = f_0(\eta)$, $f(\eta;1) = f(\eta)$ (26)

When p increases from 0 to 1 then $f(\eta;p)$vary form$f_0(\eta)$ to $f(\eta)$.By using Taylor's theorem and using (23):

$f(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)p^m$,

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m(f(\eta;p))}{\partial p^m} \quad (27)$$

$f(\eta) = f_0(\eta) + \sum_{m-1}^{\infty} f_m(\eta)$ (28)

$\mathrm{mth-order\ deformation\ equations\ are:}$ The

$$\mathrm{L}[(f_m(\eta) - X_m f_{m-1}(\eta)] = hR^f{}_m(\eta), \quad (29)$$

The boundary conditions are:

$$f'_m(0) - Knf''_m(0) = f_m(0) = f''_m(1) = 0, f''_m(0) = -1 \tag{30}$$

Where $R^f{}_m(\eta) =$
$f''' + \mathrm{Re}\sum_{i=0}^{m-1}(f_{m-1}f''_i - f'_{m-1}f'_i) + W(1 - X_m) - Mf'_{m-1} + \alpha(\sum_{i=0}^{m-1}(-f_{m-1}f''''_i +$
$2f'm-1f'''i+3f''m-1f''i))+\beta i=0m-12f''m-1f''i$ (31)

$$X_m = \begin{cases} 0 & m \le 1 \\ 1 & m > 1 \end{cases} \tag{32}$$

Tofind the solution of m th -order deformation, we shall use the symbolic software MATLAB up to first few order of approximation. We found the solution up to 2 the order approximation and they are:

f_1 =- (Re*h* η ^8)/20160 + ((Re*h)/2520 + (α*h)/2520)* η ^7 + ((α*h)/180 - (Re*h)/720 - (M*h)/720 + (β*h)/180)* η ^6 + ((M*h)/120 - (α*h)/20 - (β*h)/30 - (Re*h*kn)/120 - (α*h*kn)/60)* η ^5 + (- (Re*h*kn^2)/24 + (M*h*kn)/24 + h/24 + (W*h)/24 + (α*h)/8 + (β*h)/12)* η ^4 + η ^3/6 - η ^2/2 - kn* η

f_2 = η^7 ∗ ((α ∗ h)/5040 − (Re ∗ h)/2520 − (M ∗ h^2)/2520 − (Re ∗ h^2)/5040 + (α ∗ h^2)/336 + (β ∗ h^2)/315 + (17 ∗ α^2 ∗ h^2)/1680 + (β^2 ∗ h^2)/126 − (M^2 ∗ h^2 ∗ kn)/5040 + (3 ∗ α^2 ∗ h^2 ∗ kn)/280 + (α ∗ h ∗ kn)/1260 + (Re^2 ∗ h^2 ∗ kn^3)/5040 + (α^2 ∗ h^2 ∗ kn^2)/420 − (M ∗ W ∗ h^2)/5040 − (M ∗ α ∗ h^2)/840 − (M ∗ β ∗ h^2)/840 + (11 ∗ W ∗ α ∗ h^2)/5040 + (W ∗ β ∗ h^2)/420 − (Re ∗ h^2 ∗ kn)/5040 + (47 ∗ α ∗ β ∗ h^2)/2520 − (Re ∗ W ∗ h^2 ∗ kn)/5040 + (M ∗ α ∗ h^2 ∗ kn)/2520 + (M ∗ β ∗ h^2 ∗ kn)/420 − (Re ∗ α ∗ h^2 ∗ kn)/1680 + (Re ∗ β ∗ h^2 ∗ kn)/2520 + (11 ∗ α ∗ β ∗ h^2 ∗ kn)/1260 − (Re ∗ α ∗ h^2 ∗ kn^2)/1008 − (Re ∗ β ∗ h^2 ∗ kn^2)/420) − η^15 ∗ ((Re ∗ α^2 ∗ h^3)/70761600 − (M ∗ Re^2 ∗ h^3)/412776000 − (Re^3 ∗ h^3)/707616000 + (31 ∗ Re^2 ∗ α ∗ h^3)/1651104000 + (19 ∗ Re^2 ∗ β ∗ h^3)/1238328000 + (Re^3 ∗ h^3 ∗ kn)/990662400 − ⋯ … … … … … ….

5- Converge of solution (4)

We notice that the explicit analytical expression in eq.(29) contain the auxiliary parameter h_1.As pointed out by Liao [15] ,the convergence region and the rate of approximations given by the HAM are strongly depending on h_1.. By means of so-called h-curve for the velocity profile figure (2). The range of admissible value of $h_1$for the velocity profile is$-0.8 \leq h_1 \leq 0.8$. For the velocity distribution, tables (1) and (2) illustrate the values of the first and second derivatives for different order of the approximations . It is noted that the best value for h is-0.2.

values of h_1	f'
-0.8	-1.9831
-0.6	-1.4873
-0.4	-0.9914
-0.2	-0.49581
0.2	0.4958
0.4	0.9916
0.6	1.4873
0.8	1.9831

Table (1)the values of the convergence parameter h using the first derivative.

Values of h_1	f''
-0.8	-14.2203
-0.6	-8.9843
-0.4	-5.1173
-0.2	-2.2467
0.2	1.9953
0.4	4.1115
0.6	6.7215
0.8	10.1972

Table (2)the values of the convergence parameter h using the second derivative.

6- - Solution Using Homotopy Analysis Method

In this section HAM is applied to solve (18) subject to the boundary conditions (19) . The initial guesses and linear operators are chosen in the following :

$$\theta_\circ(\eta) = Kn - \frac{1}{2}\eta^2 \quad (33)$$

As the initial guess approximation for $\theta(\eta)$is

$L_2(\theta) = \theta''$(34)

As the auxiliary linear operator has the property:

$L(c_1 + c_2\eta) = 0$ (35)

And$c_i(i = 1 - 2)$are constant. Let p∈ [0,1]denotes the embedding parameter and h indicates non zero auxiliary parameters .Then the following equation are constructed:

$$\text{Zeroth} - \text{order deformation equations}$$

$(1 - p)L_2(\theta(\eta; p) - \theta_0(\eta)] = ph_2N_2[\theta(\eta; p)]$(36)

θ(0;p)-Kn $\theta'(1; p)$ = 0, $\theta(1; p) = 1$

(37)

$N_2[\theta(\eta; p)] = \theta''(\eta; p) - Brf''^2(\eta; p) + Pe(f(\eta; p)\theta'(\eta; p) - 2Pe(f'\eta; p)\theta(\eta; p))$=0

(38)

for p=0 and p=1:

$\theta(\eta; 0) = \theta_0(\eta)$, $\theta(\eta;1) = \theta(\eta)$ (39)

When p increases from 0 to 1 then $\theta(\eta; p)$vary form$\theta_0(\eta)$ to$\theta(\eta)$.By using Taylor's theorem and using (36):

$\theta(\eta; p) = \theta_0(\eta)$ +$\sum_{m-1}^{\infty} \theta_m(\eta)p^m$, $\theta_m(\eta) = \frac{1}{m-2!}\frac{\partial^m(\theta(\eta;p))}{\partial p^{m-2}}$

(40)

$$\theta(\eta) = \theta_0(\eta) + \sum_{m-1}^{\infty} \theta_m(\eta) \quad (41)$$

$$\text{The mth} - \text{order deformation equations}$$

L$[(\theta_m(\eta) - X_m\theta_{m-1}(\eta)] = hR^{\theta}{}_{m-1}(\eta)$, The boundary conditions are:

$\theta_m(0) - Kn\theta'_m(0) = \theta_m(1) = 0$ (43)

Where $R^{\theta}{}_{m-1}(\eta) =$
θ''_{m-1}+Br$\sum_{i=0}^{m-1}(f'''_i f'_{m-1-i})$ +Pe$\sum_{i=0}^{m-1}(f_i\theta'_{m-1-i}) - 2Pe\sum_{i=0}^{m-1}(f''_i\theta_{m-1-i})$

(44)

$$X_m = \begin{cases} 0 & m \le 1 \\ 1 & m > 1 \end{cases} \quad (45)$$

Tofind the solution of m th -order deformation, we shall use the symbolic software MATLAB up to first few order of approximation . we found the solution up to 2nd. order approximation and they are:

θ_1= *Kn - η - η ^3*((Br*h)/3 - (Pe*h*kn)/6) - η ^4*((Pe*h)/8 - (Br*h)/12 + (Pe*h*Kn)/12) + η ^2*(Pe*h*kn^2 - h/2 + (Br*h)/2) - η ^2/2 + (Pe*h* η ^5)/60 + (Pe*h* η ^6)/90*

$\theta_2 = kn - \eta + \eta$^2 $*$ $(Br*h - h/2 + (Br*h^2)/2 - h^2/2 + Pe*h^2*kn$^2 $+ 2*Pe*h*kn^2) - \eta$^3 $*((Br*h)/3 - (Pe*h*kn)/6) + \eta$^10 $*((11*Pe*Re*h^2)/604800 + (Pe*a*h^2)/37800) - \eta$^4 $*((Pe*h)/8 - (Br*h)/12 + (Pe*h*kn)/12) + \eta$^2 $*(Pe*h*kn$^2 $- h/2 + (Br*h)/2) - \eta$^3 $*((2*Br*h)/3 + (Br*h^2)/3 - (Pe*h*kn)/3 - (Pe*h^2*kn)/6) - \eta$^4 $*((Pe*h)/4 - (Br*h)/6 + (Pe*h^2)/8 + (Pe*h*kn)/6 + (Br*W*h^2)/12 + (Br*\alpha*h^2)/4 + (Br*\beta*h^2)/6 + (Pe*h^2*kn)/12 + (Br*M*h^2*kn)/12 - (Br*Re*h^2*kn^2)/12) - \eta$^9 $*((Pe^2*h^2)/5184 + (Br*Re*h^2)/12960 + (M*Pe*h^2)/10368 + (Pe*Re*h^2)/40320 - ($……..-

7- Converge of solution (6)

We notice that the explicit analytical expression in eq.(24) contain the auxiliary parameter h_2.As pointed out by Liao [15] ,the convergence region and the rate of approximations given by the HAM are strongly depending on h_2.. By means of so-called h-curve for the heat transfer profile figure (4). The range of admissible value of $h_2$for the heat rang is $-0.8 \le h_2 \le 0.8$. For the heat distribution, table (3) illustrate the values of the firstderivatives for different order of the approximations . It is noted thatthe best value for h is-0.2.

Value ofh_2	θ
-0.8	-4.3513
-0.6	-4.6346
-0.4	-5.0038
-0.2	-5.4590
0.2	-6.6270
0.4	-7.3398

0.6	-8.1386
0.8	-9.0233

8-Result and discussions:

8-1 The velocity profile :

In this section the effect of Reynolds number “Re” , the MHD parameter “M” , the Knudsen “kn” ,physical quantity at wall “W” ,and the materials of fluid “α,β”were examined

- Figure(4) shows the effect of dimensionless Reynolds number “Re” , in which the values of parameter “M” , the Knudsen “kn” ,physical quantity at wall “W” ,and the materials of fluid “α,β” are (1,0.1,1,1,2) respectively , Reynolds number “Re” is kept by values (7,8,9) the following result is obtained :when Reynolds number “Re” is increases then the velocity profile is increases too.

- In effect of parameter “M” ,the values ofdimensionless Reynolds number “Re” , the Knudsen “kn” ,physical quantity at wall “ W” ,and the materials of fluid “α,β” (7,0.1,1,1,2)respectively ,and (1,5,10) were the values of MHD parameter “M” . As MHD parameter “M” increases a decrement in the velocity profile see figure(5) .
- To study the effect of dimensionless Knudsen “kn” the values of

of Reynolds number “Re” , the MHD parameter “M” , ,physical quantity at wall “W”,the materials of fluid “α,β” were fixed (7,1,1,1,2) respectively , and dimensionless Knudsen “kn” is taken the values (0,0.1) the following results are obtained: The values of velocity increases when Knudsen “kn” increases see figure

(7)

- Figure(8,9) illustrates the effect of dimensionless parameter , β, α on the velocity profiles for fixed Re=7,M=1,kn=0.1,W=1 ,β=2,6,8 and α= 1,5 ,10 It is obvious from this figure that the effects of α , β is very strong on the velocity profile where it increases,because the value of the velocity became very small and if α =0 and $\beta = 0$,then the velocity profile would be verified by flow of Newtonian fluid cases[17].
- Figure(6) depicts the velocity for Re=7,M=1 kn=0.1,$\alpha = 1, \beta = 2$ It is obvious for this figure that the velocity decreases if the value of W is large.

8-2The heat distribution :

- Figure(14)depicts the profiles of temperature in viscoelastic fluid .when Re=7 ,M=1,α =1, β =2,that the effect of Peclet number on temperature profile is shown .According to definition of

Peclet number, increasing of Peclet number leads to increases in the temperature distribution in micro channel.

- Figure(10,11) illustrates the effect of dimensionless parameter , β, α on the heat transfer for fixed Re=7,M=1,kn=0.1,W=1 ,Pe=1and Br=1 It obvious from this figure that α , β is heavily affect on the heat transfer where it increases, and if α =0 and $\beta = 0$,then the flow fluid becomes of Newtonian [17].
- The heat transfer is fixed , when the Knudsen "kn" is increasing see figure(12).

- Figure (13) illustrates the effect ofphysical quantity at wall "W" for fixed Re=7,M=1,kn=0.1, $\alpha = 1$, $\beta = 2$,Pe=1and Br=1 It obvious that the heat is increasing when W increased .
- In effect of parameter "Br" ,we kept the values ofdimensionless Reynolds number "Re" , the Knudsen "kn" ,physical quantity at wall "W" , Pe=1and the materials of fluid "α,β" by (7,0.1,1,,11,2)respectively ,. As parameter "Br" increases there is decreasing in the heat transfer see figure 15.

9- Conclusions:

The flow of second order fluid in a micro channel is studied by Homotopy Analysis method in this paper ,and the approximate analytic solutions are obtained .The major conclusions in the research are :

1- When the fluid flow .The Reynolds number Re, the magneto number M , the Kundsen number Kn, the physical quantity at wall W, and the non Newtonian parameters α,β affect the velocity profile and heat distribution .
2- The effect of non Newtonian parameters α,β is so effective that lead to decrease on the velocity .
3- In general Kundsen number Kn in significant effect in which the resultant increment in velocity and heat transfer is very low .
4- Whentaking a gradually increased values for Pe ,this lead to increases in temperature ,according to Pe definition this may carve an equal distribution of heat at both sides of the channel .
5- Increasing Br leads to decreases in heat transfer .
6- At certain high temperature when α is taken large ,heat transfer starts decreases while when a high value of β is taken , will beheat transfer start to increase .
7- When taking increased values for physical quantity at wall W ,this lead to decreases in the velocity and increases in the heat transfer .

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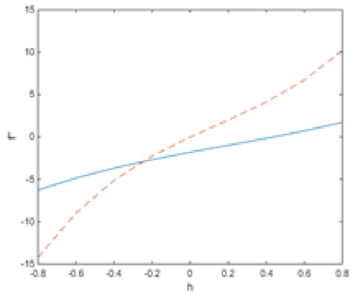


Fig.(2) the h curve for the velocity profile

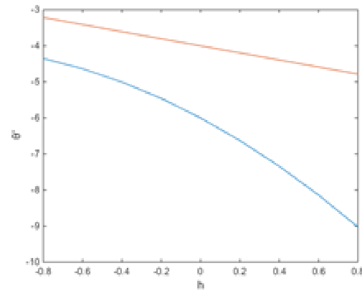
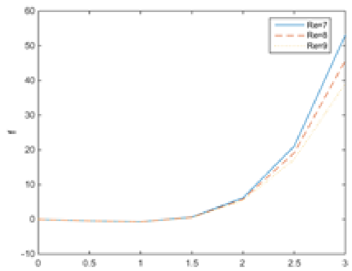
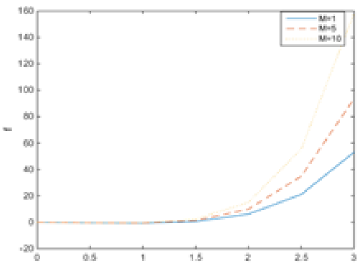


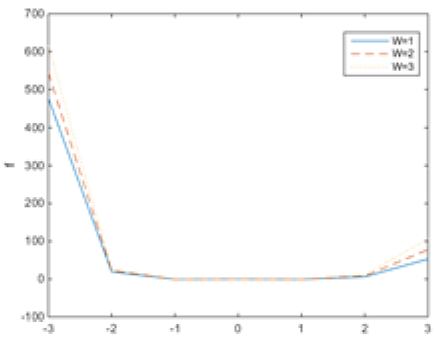
Fig.(3) the h curve for the heat transfer



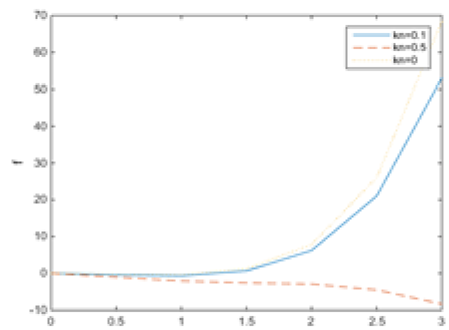
Fig(4) the effects of Re number on the velocity



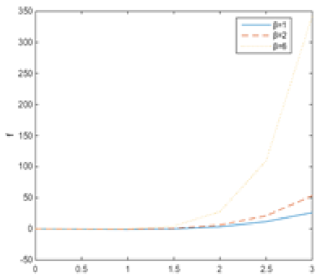
Fig(5) the effects of MDH number M on the velocity



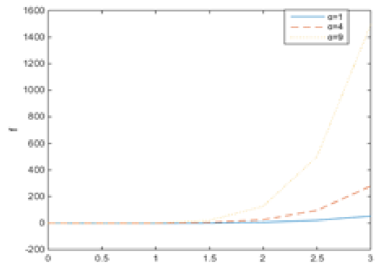
Fig(6) the effects of W at the wall on the velocity



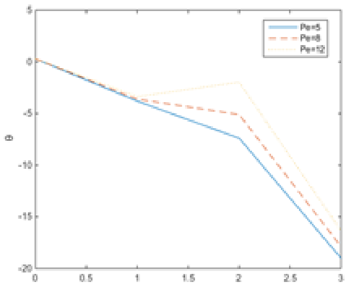
Fig(7) the effects of Kn number on the velocity



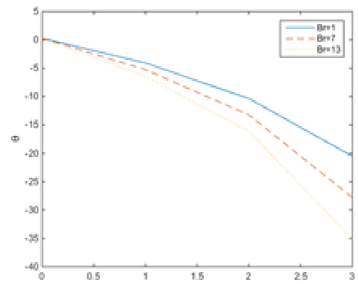
Fig(8) the effects of a parameter β on the velocity



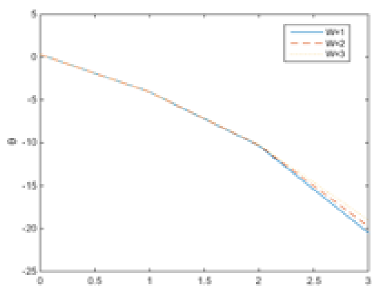
Fig(9) the effects of a parameter α on the velocity



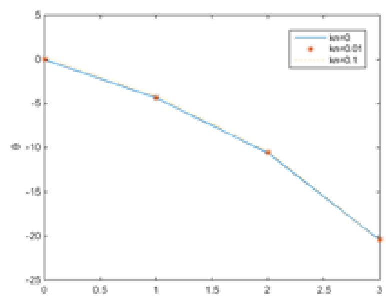
Fig(10) the effects ofPe number on the heat



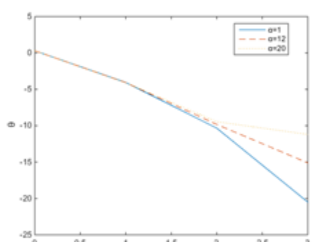
Fig(11) the effects of Br number on the heat



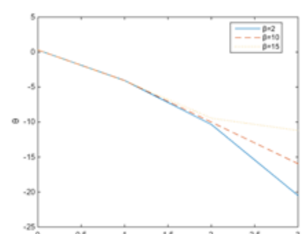
Fig(12) the effects of W on the heat



Fig(13) the effects ofKn number on the heat



Fig(14) the effects of a parameter α on the heat



Fig(15) the effects of a parameter β on the heat

Notations:

There are many symbols are used in this paper :

∇: The gradient vector

V :The velocity vector of two dimensions

u: The velocity in X direction , v: the velocity in y direction

t:The time

T:TheCaushy stress tensor

A_1, A_2: The two first Rivilin -Erilksen tensor

α_1, α_2: The material moduli of fluid

J×B: Lorenz force vector , J×B:=$\sigma u B^2$

P: The pressure ,ρ: the density

K:The thermal conductivity

C_p: The specific heat

μ: The dynamicviscosity , v: the kinematic viscosity

U: The uniform velocity ; H: The height (boundary)

Re: Reynolds number , Re= $\frac{\rho U H}{\mu}$, M: the magneto number ,M=$\frac{\sigma u B^2}{\mu}$

$\propto, \beta$: Non dimensionless parameters , $\propto = \frac{U\alpha_1}{\mu H}$, β=$\frac{U\alpha_2}{\mu H}$

W :The physical quantity at wall ,W=$\frac{H^3}{vUx} C_x$

T_0, T_1: The temperature

Pe: Peclet number , Pe=$\rho U H C_p / H$

Br:Brinkman number ,Br=$\frac{1}{\lambda} \frac{\mu U^2}{T_1 - T_0}$

Kn: Knudsen number ,Kn=2 $\frac{l}{y}$